Lattice vs. Decision Analysis
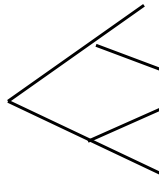
Richard de Neufville
Professor of Engineering Systems
and of
Civil and Environmental Engineering
MIT

Outline

- **Structure**
  - **Similarity – tree structure**
  - **Difference – location of decisions**
  - **Regularity, or not**
  - **Possibility of Negative Outcomes**
- **Calculations**
  - **Procedure**
  - **Size limitations**
- **Which more suitable to what situations?**

Similarity – Tree Structure

- **Both Lattice Model and Decision Analysis have Tree Structure**

- **What are differences?**
  - Shape: Lattice: 2 to 2 to 2….   DA: unlimited
  - Order: Lattice: states only
       DA: decision – outcome – decision -- outcome

Dealing with Decisions

- **How does Lattice Model include decisions?**

- **"one at a time" is routine (e.g.: 'close mine')**
  - Anything else requires special treatment
  - Two decisions simultaneously?   Can do, but…

- **Sequence of decisions (e.g.: close, then open)?**
  - GREAT DIFFICULTY
  - Path dependence the problem

Path Independence:
Implicit Assumption of Lattice Analysis
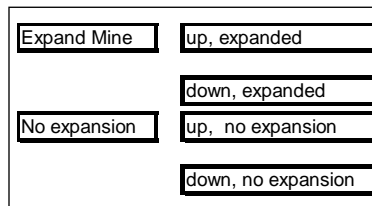
Pay Attention – Important point often missed!

- **Model Implicitly assumes "Path Independence"**
  - **Since all paths to a state have same result**
  - **Then value at any state is independent of path**
  - **In practice, this means nothing fundamental happens to the system (no new plant built, no R&D , etc)**

When is "Path Independence" OK?

- **Generally for Financial Options (stocks, Foreign exchange, commodities)   Why?**
  - **Random process, no memory….**

- **Often not for Engineering Systems. Why?**
  - **If demand first rises, system managers may expand system, and have extra capacity when demand drops.**
  - **If demand drops then rises, they won't have extra capacity and their situation will differ**
  - **Process – and result -- then depends on path!**

Example for "path independence"

- **Suppose you expand the mine in year 3…**
- **What is situation in later stages?**

- **In year 4?**

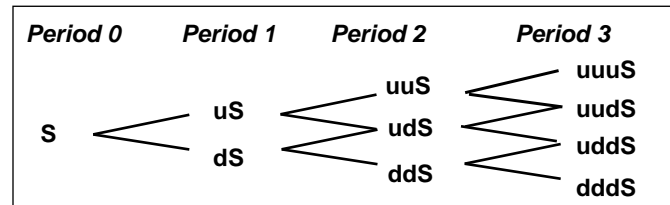| Expand Mine	up, expanded
	down, expanded
No expansion	up, no expansion
	down, no expansion

2 different physical states of system for same outside state (of price)

- **In year 5? More combinations in 3 dimensions**

Regularity of Binomial Model

- **Lattice Model assumes diffusion process is "stationary"**
    - **probability of next states stays the same throughout periods considered**

- **Decision Analysis not limited this way**
    - **Probabilities can differ after decisions in a stage**
    - **… and for each stage**
    - **Ex: P(Environmental penalty) could depend on**
        - * **Decisions made by industry now**
        - * **Changes in government in later stages**

Binomial Lattice: Several periods

Period 0	Period 1	Period 2	Period 3

S ⟶ uS, dS ⟶ uuS, udS, ddS ⟶ uuuS, uudS, uddS, dddS

- **Process continues identically throughout period being considered**

Possibility of Negative Outcomes

- **Lattice does not allow <u>states</u> to shift from positive to negative values:**
  - **Sign of extreme values ($d^n S$, $u^n S$) same as S**

- **This is realistic only for factors which cannot be negative – such as price**

- **Lattice <u>does permit negative outcomes</u>, when value model transforms positive state (Cu price) to negative amount (mine profits)**

- **Decision Analysis has no limitation**

Summary of Structural Comparison

Characteristic	Decision Analysis	Lattice Approach
Structure Nature	Tree	Tree
Structure Complexity	As desired	Binomial
Location of Decisions	For each state and stage independently	Brought in stage by stage from end
Number of Decisions	As many as desired	More than one is difficult
Regularity of Process	None assumed	Stationary Process
Negative Outcome	Anywhere	Inserted through value model

Decision Analysis clearly is the more flexible approach

Which Approach Most Suitable?

- **What do you think?**

- **Depends on Circumstances**

Problem Characteristic	Decision Analysis	Lattice Approach
Steady Change		Better
Sudden Changes, "jumps"	Better	
One Decision		Better
Many Decisions	Better	

- **What if mixed circumstances?**

Differences – Set up for Analysis

- **Lattice Model uses a repetitive process (a recurrence formula) that is similar from stage to stage, and between states.**
  - **Simple modular process, easy to program**

- **Decision Analysis can be different at each stage and step**
  - **DA Programs available (Crystal Ball, Treeage, etc)**
  - **Detailed set up required (input of vectors of outcomes, probabilities) that can differ**

Differences – Calculation Time

- **Lattice Model**
  - **Problem Size proportional to N, number of stages**
  - **Can easily consider 100s of stages**
  - **However, looking at only 1 decision (to use flexibility or not).**
  - **Thus results understandable even for large lattice**

- **Decision Analysis**
  - **Problem Size proportion to power of N**
  - **Looking at more than a few stages becomes complicated – and unintelligible to user**
  - **3 stages is the most I've seen used effectively**

Summary of Analytic Comparison

Characteristic	Decision Analysis	Lattice Approach
Analysis Concept	Right to left, end to start	Right to left, end to start
Set up	"hand-crafted"	Standard Module
Number of Periods	In practice, 3 is maximum	Hundreds
Intelligibility	Not obvious	Identifies single decision

Decision Analysis clearly is the more difficult approach

Summary

- **Lattice Model and Decision Analysis are similar ways of investigating flexibility**

- **Each has its own role:**

  Lattice model better for single decisions over many stages

  Decision Analysis better for complex, irregular processes over couple of stages