Hybrid Lattice and Decision Analysis

Based upon Master of Science Thesis
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Purpose

- Analyze options in systems where 2 types of uncertainties exist simultaneously:
  - Small, incremental uncertainties – evolve in continuous time steps in a binomial lattice
  - Large, step uncertainties – evolve in discrete "jumps" and are typically analyzed with decision trees

Examples of Two Uncertainty Types in Systems

System	Incremental Uncertainty	Large Step Uncertainty
Airport	General regional traffic	Airline hub decision
Energy Sector	Number of households	Government policy
Business Expansion	Demand for a product	Competitor entrance into market

Advantages of Hybrid Analysis

- Accounts for path dependency
  - While maintaining lattice recombination
- Presents decisions in terms of distributions
  - Not simply as single value descriptors such as expected value
- Allows managers to express their risk preferences directly
  - Without the need for utility theory or an assumption of risk-neutrality

Main Disadvantage of Hybrid Analysis

- Inherits “curse of dimensionality” from decision trees
  - Must limit number of options and time periods: thesis outlines procedure for 1 option over 2 periods in the decision tree
  - May require other simplifying assumptions to maintain feasibility of sensitivity analysis

Steps of the Hybrid Method

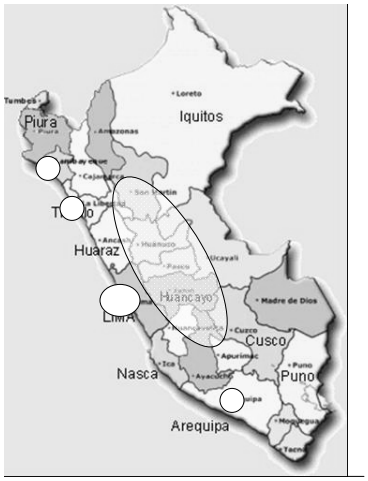
1. Build the decision tree and identify scenarios.
   (Scenario = a unique path through the tree)
2. Construct the lattice Target Curves for each scenario from t=0 point of view.
3. Combine Target Curves at tree chance nodes.
4. Decide among a set of combined Target Curves at decision nodes.

Steps of the Hybrid Method (2)

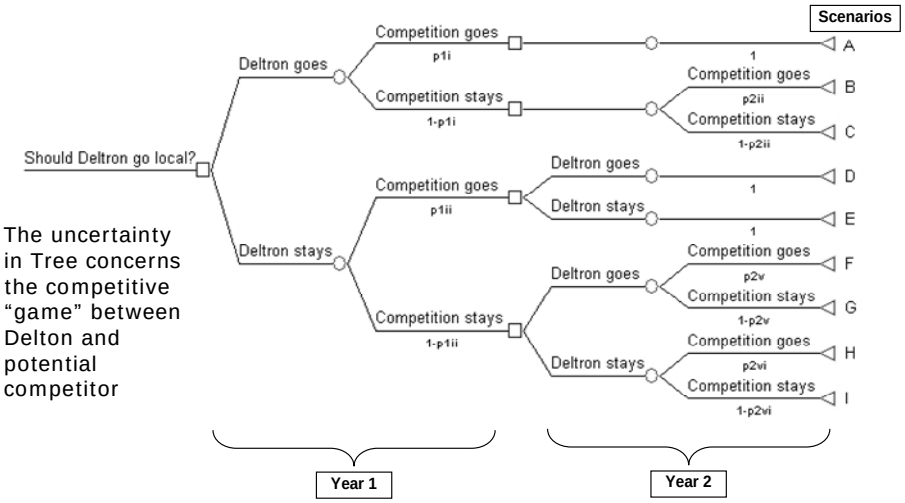
5. Repeat steps 3 and 4 over the first period, working back to the initial tree node.
6. Repeat entire procedure after first period in tree elapses.

About the hypothetical case

- Grupo Deltron is a leading computer wholesale distributor in Peru
- Demand growing in interior of the country
  - Opportunity to set up a Local Distribution Mode



Step 1: Build Decision Tree and Identify Scenarios



The uncertainty in Tree concerns the competitive "game" between Delton and potential competitor

Step 1 (2): Build Decision Tree and Identify Scenarios

- Parallel to tree development (i.e, actions of Deltron and competitors), the regional demand for PCs evolves through a lattice over the 2 years.

v = 15% / yr

σ = 25% / yr

p = 71%

u = 1.19

d = 0.84

ORIGINAL OUTCOME LATTICE				
6 Month Periods				
0	1	2	3	4
25000	29834	35603	42487	50703
	20949	25000	29834	35603
		17555	20949	25000
			14710	17555
				12327

Step 2: Construct the lattice Target Curves for each scenario

- Example: Scenario B – Deltron goes local now, but competition waits until year 2 to do the same
  - Competition's entrance is modeled by **multiplicative factor z** – a % decrease in effective demand for Deltron PC's
  - Use of this factor **z** allows for path dependency while maintaining lattice recombination

Step 2 (2): Construct the lattice Target Curves for each scenario

EFFECTIVE OUTCOME LATTICE				
6 Month Periods				
0	1	2	3	4
25000	29834	35603	31865	38027
	20949	25000	22376	26702
		17555	15712	18750
			11033	13166
				9245

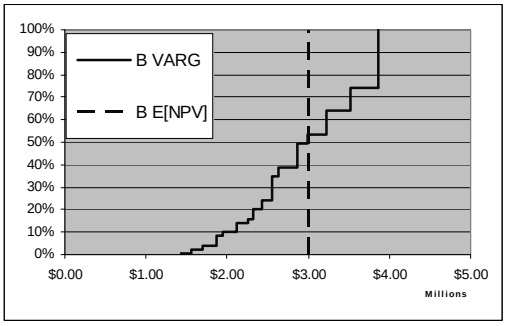
In Year 2 (Periods 3 and 4) the Original Outcome Nodes undergo a z=25% reduction.

EFFECTIVE INSTANT VALUE LATTICE				
6 Month Periods				
0	1	2	3	4
-$125,000	$1,002,293	$1,217,714	$1,078,149	$1,308,238
	$670,510	$821,776	$723,775	$885,341
		$543,753	$474,938	$588,388
			$300,207	$379,871
				$233,452

Each effective demand node translates into an undiscounted dollar value, through a benefit function.

Step 2 (3): Construct the lattice Target Curves for each scenario

- The Effective Instant Value Lattice has $2^4=16$ possible paths, each with a possibly unique NPV and probability.
  - Sorting these, one builds a Target Curve from the viewpoint of t=0 assuming scenario B occurs.



in $M	Min	Max	Spread	E[NPV]	Coefficient of Variation
B	$1.43	$3.86	$2.43	$3.00	23%

Step 3: Combine Target Curves at tree chance nodes

- How can we combine distributions?

Let A_i be the area of a probability density function for a scenario i.

By definition, $A_i = 1$.

Let p_i be the objective probability of a scenario i.

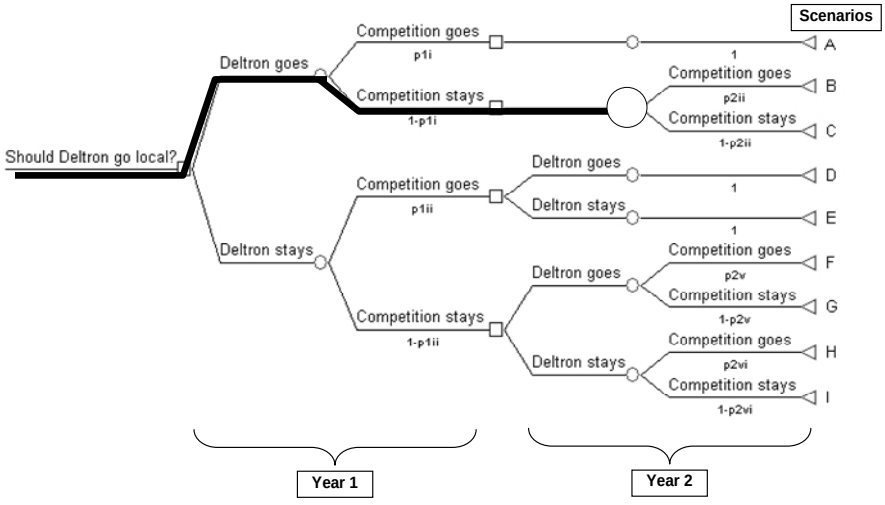
$$\text{By definition } \sum_{i=1}^{n} p_i = 1.$$

$$\therefore \sum_{i=1}^{n} p_i A_i = \sum_{i=1}^{n} p_i\,(1) = \sum_{i=1}^{n} p_i = 1.$$

Step 3 (2): Combine Target Curves at tree chance nodes

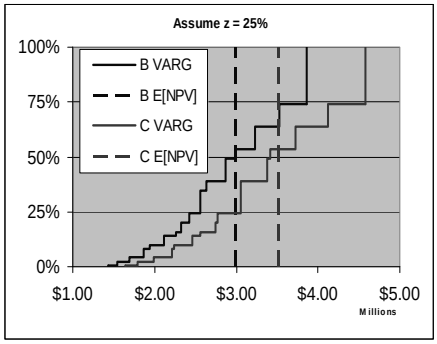
- Example: Year 2 tree chance node – Scenario B or C
  - Given that Deltron goes local in year 1 and that the competition stays, this is the chance that the competition in year 2 also goes local (B) or stays (C).

Step 3 (3): Combine Target Curves at tree chance nodes

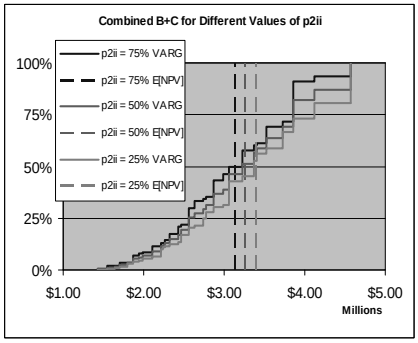


Step 3 (4): Combine Target Curves at tree chance nodes

Uncombined B and C Target Curves



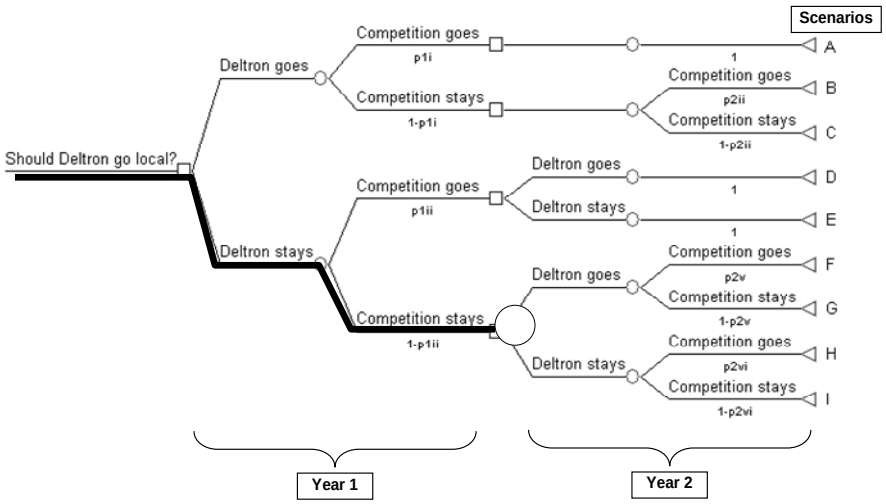
Combined B and C Target Curves: for different values of the objective probability



Step 4: Decide among combined Target Curves at decision nodes

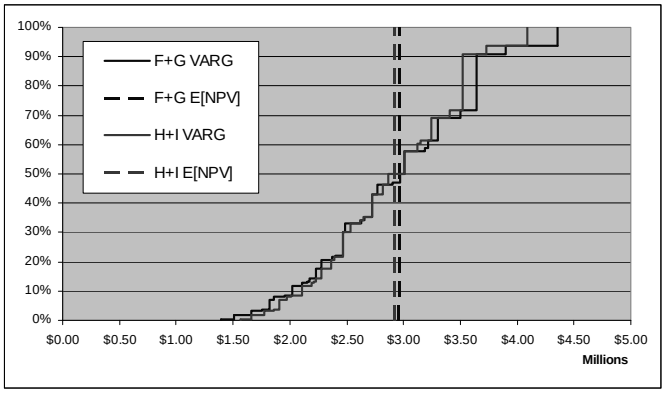
- Example: Year 2 tree decision node for F+G or H+I
  - Given that Deltron and the competition stay in year 1, Deltron must decide either to go (F+G) or to stay (H+I) in year 2.
  - Should test the decision for different values of objective probabilities and z.
    - This requires simplifying assumptions so that the sensitivity analysis is not too complicated.

Step 4 (2): Decide among combined Target Curves at decision nodes



Step 4 (3): Decide among combined Target Curves at decision nodes

Assuming z = 25%; Prob of F = Prob of H = 75%



Step 4 (4): Decide among combined Target Curves at decision nodes

Performing a sensitivity analysis for z and the objective probabilities will change the shapes of the distributions.

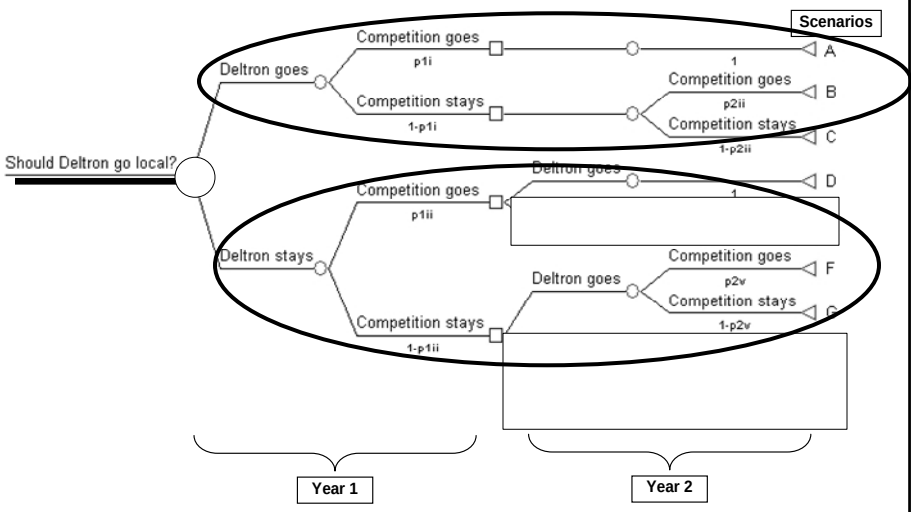
$ in M		Minimum	Maximum	Spread	E[NPV]	Coefficient of Variation
z=10%; $p_{2v}=p_{2vi}=50\%$	F+G	$1.52	$4.35	$2.84	$3.20	23%
	H+I	$1.66	$4.09	$2.42	$3.11	21%
z=25%; $p_{2v}=p_{2vi}=75\%$	F+G	$1.39	$4.35	$2.96	$2.96	24%
	H+I	$1.56	$4.09	$2.52	$2.93	22%
z=40%; $p_{2v}=p_{2vi}=99\%$	F+G	$1.27	$4.35	$3.08	$2.73	27%
	H+I	$1.46	$4.09	$2.62	$2.74	23%

So which to choose? F+G or H+I? Depends. It is entirely up to managers to **weigh pros and cons and make a direct decision based on risk preferences**.

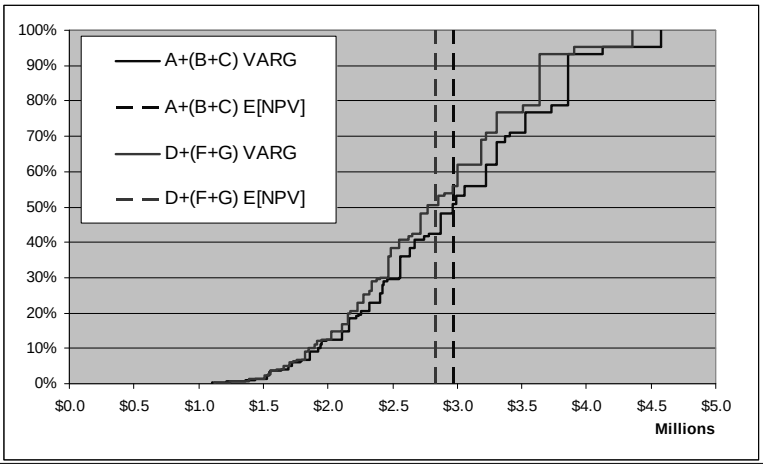
Step 5: Repeat Steps 3 and 4 to arrive at initial decision node

- Procedure for combining already combined graphs is same as before.
- Initial decision node will choose between 2 twice-combined Target Curves.
  - Example: Assume that D has been chosen over E and that F+G has been chosen over H+I.
    - i.e., going in year 2 if have not already done so is preferred regardless of what competition has done in year 1.

Step 5 (2): Repeat Steps 3 and 4 to arrive at initial decision node



Step 5 (3): Repeat Steps 3 and 4 to arrive at initial decision node



Step 5 (4): Repeat Steps 3 and 4 to arrive at initial decision node

- Choose between A+(B+C): going now or D+(F+G): not going now

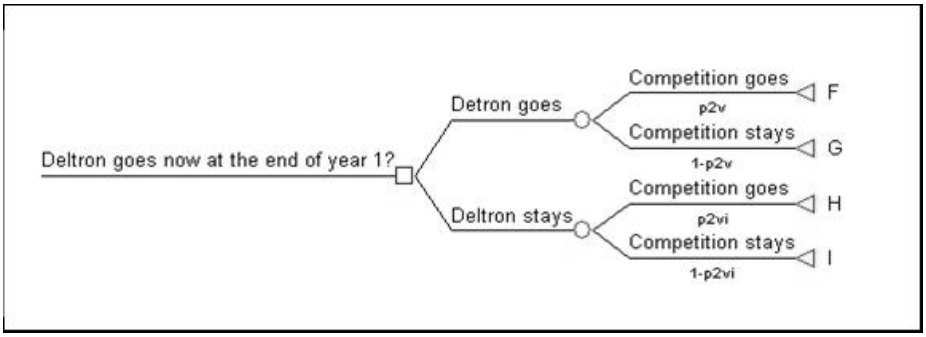
$ in M		Minimum	Maximum	Spread	E[NPV]	Coefficient of Variation
z=10%; $p_{1i}=p_{1ii}=1\%$	A+(B+C)	$1.43	$4.57	$3.15	$3.36	23%
	D+(F+G)	$1.41	$4.35	$2.94	$3.19	23%
z=25%; $p_{1i}=p_{1ii}=25\%$	A+(B+C)	$1.10	$4.57	$3.47	$2.97	27%
	D+(F+G)	$1.13	$4.35	$3.23	$2.83	26%
z=40%; $p_{1i}=p_{1ii}=50\%$	A+(B+C)	$0.78	$4.57	$3.79	$2.40	36%
	D+(F+G)	$0.84	$4.35	$3.51	$2.31	34%

Argument for going now looks stronger than that of staying now, especially comparing Target Curves. However, **final decision is purely subjective but based on objective data.**

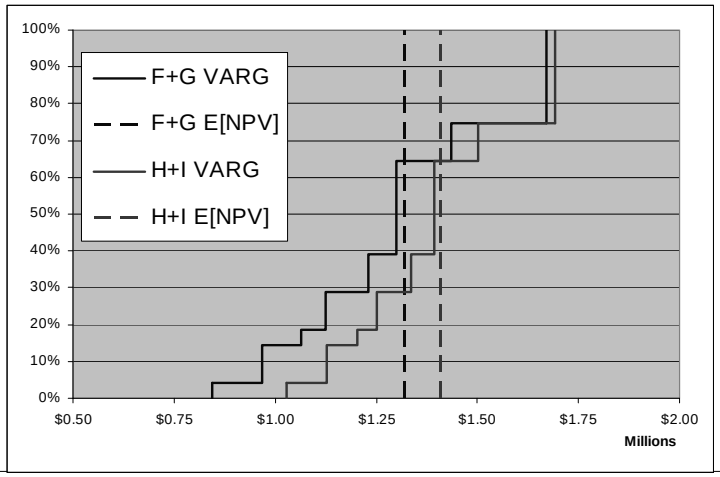
Step 6: Repeat entire procedure after first period in tree

- Suppose we are now at the beginning of year 2. Neither Deltron nor competition have gone local.
- Instead of growing by 15%, demand has actually dropped by 30%.
- How does this change previous decision to go at year 2 regardless of what competition has done in year 1?

Step 6 (2): Repeat entire procedure after first period



Step 6 (3): Repeat entire procedure after first period



Step 6 (4): Repeat entire procedure after first period

$ in M		Minimum	Maximum	Spread	E[NPV]	Coefficient of Variation
z=10%; $p_{2v}=p_{2vi}$=25%	F+G	$0.98	$1.67	$0.70	$1.45	17%
	H+I	$1.13	$1.69	$0.56	$1.51	13%
z=25%; $p_{2v}=p_{2vi}$=50%	F+G	$0.84	$1.67	$0.83	$1.32	20%
	H+I	$1.03	$1.69	$0.66	$1.41	15%
z=40%; $p_{2v}=p_{2vi}$=75%	F+G	$0.71	$1.67	$0.96	$1.09	29%
	H+I	$0.92	$1.69	$0.77	$1.23	21%

This time, decision to go in year 2 is definitely undesirable.

Conclusions for Hybrid Method

- Works well to look at a top level strategic decision over 2 periods.
- Target Curves provide ease of explanation to decision makers.
- Path dependency conflict between Lattice and Decision Analysis solved.
- If looking to solve multiple options over many periods and/or are concerned about very detailed sensitivity analysis, this is not the method for you.

Contact Information

- Thesis available at ardent.mit.edu/real_options/Real_opts_papers/Nestor_Hubrid_Lattice_and_Decision_Analysis Thesis (2).pdf

- Comments, questions, suggestions? please email Nestor Quispez-Asin: pacific@deltron.com.pe