FLAW OF AVERAGES

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Outline

- What is the concept?
- Why is it important in practice?
- When does it occur?

- How to avoid

THE CONCEPT

Flaw of Averages

- Presentation explains a fundamental problem in the design and evaluation of systems
- This problem is the pattern of designing and evaluating systems based on the “average” or “most likely” future projections
- Problem derives from misunderstanding of probability and systems behavior, known as

FLAW OF AVERAGES

Flaw of Averages

- Named by Sam Savage ("Flaw of Averages, Wiley, New York, 2009)

It is a pun. It integrates two concepts:

- A mistake => a "flaw"
- The concept of the "law of averages", that that things balance out "on average"
- Flaw consists of assuming that design or evaluation based on "average" or "most likely" conditions give correct answers

Mathematics of Flaw

- Jensen's law:
- E [f(x)] ≤ f [E(x)] if f(x) is convex function
- Notation: E(x) = arithmetic average, or "expectation" of x
- In words:
  - E[f(x)] = average of possible outcomes of f(x)
  - f [E(x)] = outcome calculated using average x

Example

Given: $f(x) = \sqrt{x} + 2$

And: x = 1, 4, or 7 with equal probability

- $E(x) = (1 + 4 + 7) / 3 = 4$
- $f[E(x)] = \sqrt{4} + 2 = 4$
- $f(x) = 3$, 4, or $[\sqrt{7} + 2] \sim 4.65$
  with equal probability
- $E[f(x)] = (3 + 4 + 4.65) / 3 \sim 3.88 \leq 4 = f[E(x)]$

More generally…

$E [f(x)] \neq f [E(x)]$

Example:

- Given: $f(x) = x^2 + 2$
- And: x = 1, 2, or 3 with equal probability
- $E(x) = (1 + 2 + 3) / 3 = 2$
- $= 4 + 2 = 6$
- $f(x) = 3$, 6, or 11 with equal probability
- $E[f(x)] = (3 + 6 + 11) / 3 = 6\frac{2}{3} \neq 6 = f[E(x)]$

When equal?

E [f(x)] = f [E(x)] when f(x) linear

This is rarely the case!

Example:

- Given: f(x) = x + 2
- And: x = 1, 2, or 3 with equal probability
- E(x) = (1 + 2 + 3) / 3 = 2
- f[E(x)] = 2 + 2 = 4
- f(x) = 3 , 4, or 5 with equal probability
- E[f(x)] = (3 + 4 + 5) / 3 = 4 = f[E(x)]

In Words

- Average of all the possible outcomes associated with uncertain parameters,
- generally does not equal
- the value obtained from using the average value of the parameters

Practical Consequences

Because Engineering Systems not linear:

- **Unless you work with distribution, you get wrong answer**
- **answer from a realistic description differs – often greatly – from the answer you get from average or any single assumption**
- **This is because gains when things do well, do not balance losses when things do not (sometimes they're more, sometimes less)**

WHY IMPORTANT IN PRACTICE

Why does Flaw occur?

Flaw is a pattern in systems design. Why?

Several reasons:

- Management fixes design parameters, thus limiting designers
- Designers deliberately choose simplicity
- Need, desire to focus on a single scenario, given limited design resources
- Professional desire for 'certainty'

Management fixes parameters

Designers often constrained by management or client to focus on one scenario

- Management specifies price of product (copper, oil) => opportunities that might be valuable for a higher price are neglected
- Client tells designers to work toward a forecast (1 M customers for Iridium…)
- Client specifies performance requirements (a different way of defining forecast needs), this is typical for military…

Designers choose simplicity

Designers often choose single scenario, even when possible ranges are available

- Example: design of oil platforms based on "best estimate" (= P50) of "oil in place"
  - Recall data on variability of estimates of oil reserves (uncertainty presentation)
  - Geologists present ranges on their estimates (P10 to P90, for example). Uncertainty is clear
  - Yet design process focus on a single number!

Need, desire to focus on 1 scenario

Detailed design of a system (automobile, oil platform) requires great effort. Yet limited time and resources available

- Hard to create one design => desire not to design many systems for different scenarios
- Computer capabilities have changed, but practice has not adapted (yet)
- Desire to "optimize" => single scenario (Example: design of Iridium satellite system)

Desire for certainty

Engineering practice often more comfortable with certainties, precision

- Tradition of scientific rigor, desire for precision – not immediately compatible with vagueness, uncertainty
- A deep issue: designers have selected engineering because it offers rigor, and they feel uncomfortable with vagueness…

Practical Consequences

Organizational, personal resistance to recognizing and dealing with uncertainty

- Not current practice for
  - Much management of design work
  - Many client relationships
- Uncomfortable personally for many individuals

Although forecast uncertainty demonstrably great, this reality is resisted

How issue arises in practice

- In practice, design rarely focuses specifically on “average” future conditions
- If you do not recognize distribution of uncertainly, you cannot calculate an average
- Focus typically on “most likely” scenario.
- Formally, not the same as “average”
- Conceptually equivalent however. Mental model is Normal distribution around best estimate, so “most likely” = “average”

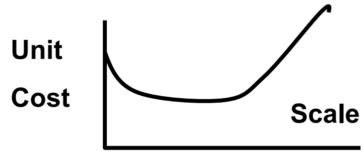
REASONS FOR SYSTEM NON-LINEARITY

3 Reasons for Non-Linearity

- System response is non-linear
- System response involves some discontinuity (step change)
- Management rationally imposes a discontinuity

System Response is Non-Linear

- Economies of Scale: Unit costs decrease with scale of production
- Large initial costs prorated over volume, so that unit costs decrease as scale increases toward capacity
- Increasing marginal costs as scale increases (labor, material costs higher)



This is Usual Situation!

System involves Discontinuity

Discontinuities = special form of non-linearity

Discontinuities are Common:

- Expansion of a Project might only occur in large increments (new runways, for example)
- A System may be capacity constrained, so that profitability or values increases with demand up to a point, and then levels off

Management Creates Discontinuity

- Managers or System Operators may decide to take some major decision about a project …
- to enlarge it or change its function
- this creates a step change in the performance of the system.
- This can happen often – and does!

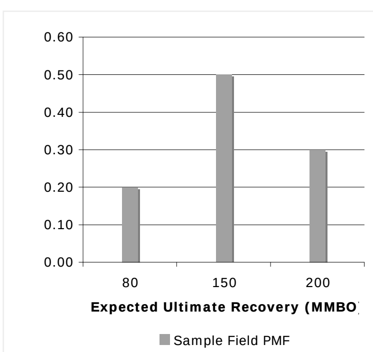
A practical example

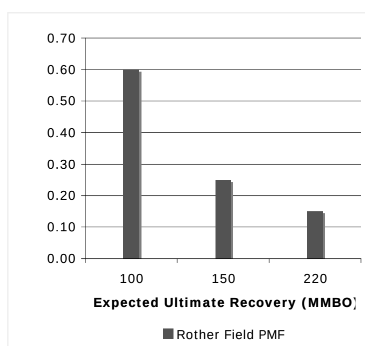
Design of oil platform and wells, Golf of Mexico

Reference: Babajide, A., de Neufville, R. and Cardin, M.-A. (2009) "Integrated Method for Designing Valuable Flexibility in Oil Development Projects," Paper 122710-PA, Society of Petroleum Engineers, Projects, Facilities and Construction, Vol.4, no. 2, June 2009

http://www.spe.org/ejournals/jsp

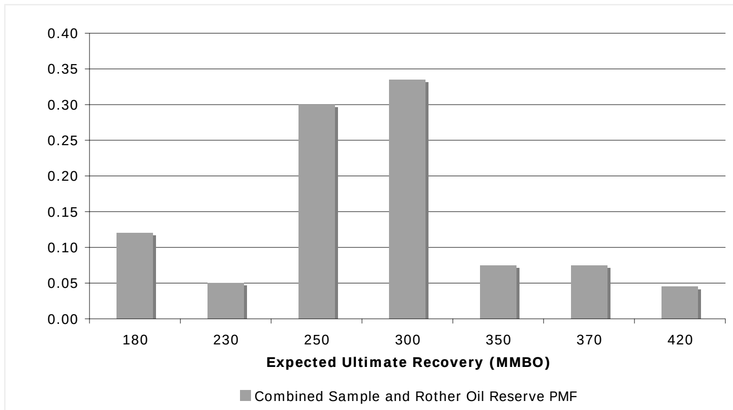
Gulf of Mexico Platform Probability Mass Functions



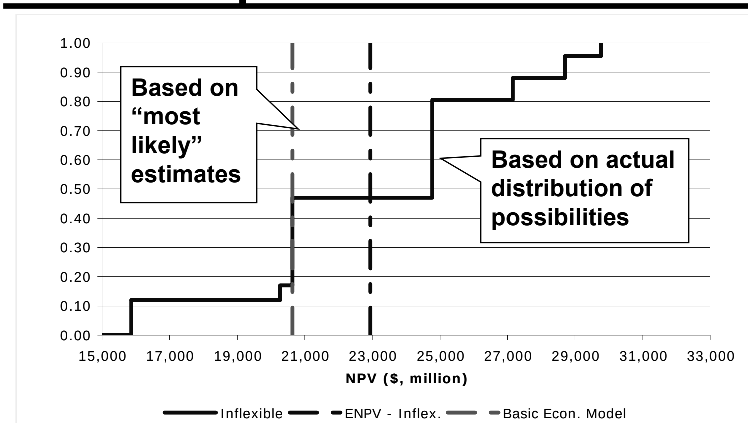


Note: "Most likely" scenarios are 150 and 100

Combined PMF



Comparison of Values



Actual ENPV $\neq$ Value based on Mostly Likely Conditions

Take-Aways

- Do not be a victim of Flaw of Averages
- Do not value projects or make design decisions based on average or most likely forecasts – your results will be WRONG
- Do consider the range of possible events and examine distribution of consequences
- This will be hard – standard paradigm locks on single, “best” estimates. Shift to new, correct paradigm is difficult for many.