Lattice Valuation of Options

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Outline

- Uncertainty manifested in evolution of
  - Outcomes of uncertain process,
  - Probabilities associated with these outcomes
  - Impacts on system of these uncertain outcomes
- Integrating Elements of System
- Analysis of Value of Option
  - Principle: a multi-stage decision analysis
  - Practice: examples "on" and "in" systems
- Graphical Illustration of results

Manifestations of Uncertainty (1)

Three elements part of valuation of options:

1. **The uncertain process that generates a range of possible outcomes, for example**
   - Demand or Price of product
   - Quantity or Quality of product
   - Tax Regime, Environmental Regulations, etc.

- **Usual to assume that range expands as we project farther into future**

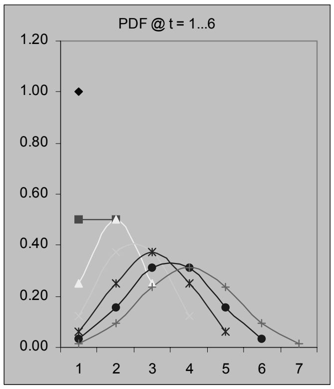
Manifestations of Uncertainty (2)

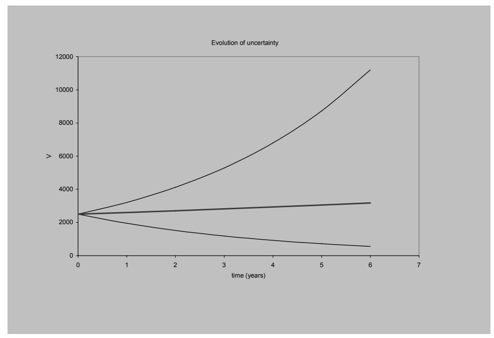
2. **Probabilities associated with outcomes, that is, the chance that a state is achieved**

- **Usual to assume these probabilities**
  - Part of a Diffusion process (as for lattice projection) , so over time they
  - Increase for more extreme outcomes
  - Decrease for central outcomes
- **Alternatives possible**
  - Example: Probability of any state constant…

Manifestations of Uncertainty (3)

The diffusion of probabilities in pictures





Graphs from K Konstantinos

Manifestations of Uncertainty (3)

3. Impacts on system, that is, the effects of the uncertain outcomes on system performance. For example, how
   - Demand or Price impacts profitability
   - Quantity or Quality determines performance
   - Tax Regime, Environmental Regulations, etc alter the efficiency of a system.

- Models required to translate outcomes of uncertain process into system performance

Integration of Elements

- Any uncertain outcome …
  Influences the performance of the system
- The PDF of the uncertain outcomes…
  leads to another, different PDF, of system performances which may be
  - Automatic (no system management), or
  - Shaped by intelligent control: System Managers take advantage of flexibility to adapt system to uncertain environment

Example Integration of Elements

- The technological system is a copper mine…
- The uncertainty concerns price of copper…
- Profits depend on price of copper -- but not linearly, because there are large fixed costs and variable operating costs…
- Operators can use flexibility to shape profits
  - Close mine if prices low; expand if prices high
  - Alter "mine plan" to allocate digging operations most effectively between exploiting rich deposits and getting rid of sterile overburden

Valuation of Flexibility

- The question before the system managers:
  "What is the value of the flexibility?"
- When they can answer this, they will know if:
  - value of flexibility > cost of acquiring it
  - Flexibility should be designed into system
- The analytic question is:
  "How do we value the flexibility?"

Valuation Process (general)

- The value of flexibility – of options – is defined like value of information
- It is an Expected Value
- Decision Analysis of situation without added flexibility gives "base case" expected value
- DA incorporating flexibility gives new EV
- Value of Flexibility is the difference

Valuation Process (in detail)

- Lay out the possible states over all periods
- With their associated probabilities
- From perspective of last period:
  - knowing the value of the possible results – calculate the expected value of the best choice
  - This is the value for the beginning of that period
  - Repeat process until start of 1st period, which gives expected value of tree
- Two demonstrations follow…

The example situation

- A copper mine… producing 5000 tons/year
- Control for 6 periods
- Revenue/period = 5000 x (price, end of period)
- Current price is $2000/ton and we suppose
  - Average price increase, v = 5% / year
  - Standard deviation, σ, is 10%
- The annual $ costs of operating the mine are:
  - 1,000,000 + 2,200(tons produced)
- Discount rate is 12%

Evolution of Price -- parameters

- **The historic data enable us to project the evolution of the copper prices**
- **First we calibrate p, u, d (see lattice slides)**

p	0.75	$0.5 + 0.5\ (v/\sigma)\ (\Delta t)^{0.5}$
u	1.105171	$e\ \exp[\ (\sigma)\ (\Delta t)^{0.5}]$
d	0.904837	1/u

Price Evolution – States and Probabilities

To get (here from binomial lattice.xls):

PROBABILITY LATTICE

1.00	0.75	0.56	0.42	0.32	0.24	0.18
	0.25	0.38	0.42	0.42	0.40	0.36
		0.06	0.14	0.21	0.26	0.30
			0.02	0.05	0.09	0.13
				0.00	0.01	0.03
					0.00	0.00
						0.00

OUTCOME LATTICE

2000	2210	2443	2700	2984	3297	3644
	1810	2000	2210	2443	2700	2984
		1637	1810	2000	2210	2443
			1482	1637	1810	2000
				1341	1482	1637
					1213	1341
						1098

Impact of Uncertainty on System

- **Each state of uncertainty affects system**
  - Here: price of copper in any year affects revenues

Revenue = Tons(price) – (Fixed Cost) – Tons(2200)

= 5000 (price-2200) – 1,000,000

- **=> lattice of revenues (losses in red):**
  - most entries red, but their probabilities are low

2,000,000	948,291	214,028	1,498,588	2,918,247	4,487,213	6,221,188
	2,951,626	2,000,000	948,291	214,028	1,498,588	2,918,247
		3,812,692	2,951,626	2,000,000	948,291	214,028
			4,591,818	3,812,692	2,951,626	2,000,000
				5,296,800	4,591,818	3,812,692
					5,934,693	5,296,800
						6,511,884

How bad is this project?

- **Quick look at possible outcomes makes project look terrible…**
- **HOWEVER, PDF is skewed toward success – probability of losses quite small… (slide 14)**
- **Here is picture of [probability x revenues] which shows contribution to expected value**

2,000,000	711,218	120,391	632,217	923,352	1,064,837	1,107,238
	737,906	750,000	400,060	90,293	592,703	1,038,771
		238,293	415,072	421,875	250,038	63,487
			71,747	178,720	259,420	263,672
				20,691	67,263	125,662
					5,796	23,277
						1,590

Base Case Value of System

- Base Case assumes no flexibility
  - Production is "automatic" -- it continues even if price low. (Might be required by contract)
- Annual revenues = Sum of yearly columns

Annual Expected Rever	1,449,125	867,903	254,663	392,359	1,075,023	1,795,294
NPV	$398,112					

Notes to Calculation of Expected NPV:

1. Assuming Revenue in any year depends of end of year price, initial $2000 price is not used
2. Annual expected revenues discounted at 12%

Flexibility "on" the System

- Assume system operators can close mine permanently in any year.
- This is an put option "on" the system
  - An "option" because it is "right, not obligation" to change operations
  - A "put" because it gets operators out of losses
  - "on" system, because it does not change technology of system
- What is the value of this option?

Decision to Exercise Option

- **When to exercise option is NOT OBVIOUS!**
- **Consider evolution of possible revenues… Should operator close in 1st year because of possible loss?**
- **Not clear! Good chance of big recovery!!**

2,000,000	948,291	214,028	1,498,588	2,918,247	4,487,213	6,221,188
	2,951,626	2,000,000	948,291	214,028	1,498,588	2,918,247
		3,812,692	2,951,626	2,000,000	948,291	214,028
			4,591,818	3,812,692	2,951,626	2,000,000
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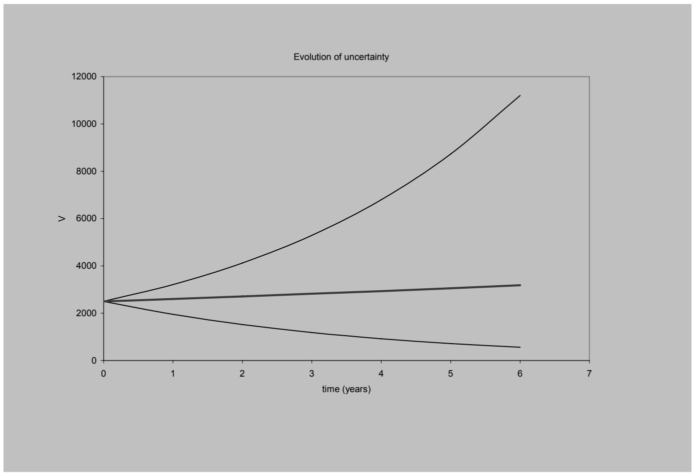
Non-convexity of Feasible Region

- **Note carefully – in general, the feasible region is not convex**
  - As evolution of system follows upward bending exponential growth (slide 5, repeated next)
- **This has an important consequence:**
  - Looking at marginal conditions (also known as "myopic rule") is not sufficient
  - Distant, longer-run may overtake short-run losses
  - See later presentation on Dynamic Programming

Non-convexity of Feasible Region



Graph from K Konstantinos

Analysis of Decision to Exercise

- To simplify, we restate revenues in millions

2.00	0.95	0.21	1.50	2.92	4.49	6.22
	2.95	2.00	0.95	0.21	1.50	2.92
		3.81	2.95	2.00	0.95	0.21
			4.59	3.81	2.95	2.00
				5.30	4.59	3.81
					5.93	5.30
						6.51

Note: red figures conventionally indicate losses

- We now analyze as with decision tree…
- For example, suppose we at the end of the 5th year with the worse prices (boxed cell) – what would our decision be?

Decision at a particular state

- From this state, prospects for last (6th) year are losses > closed mine: 5.30 > 1 ; 6.51 > 1

2.00	0.95	0.21	1.50	2.92	4.49	6.22
	2.95	2.00	0.95	0.21	1.50	2.92
		3.81	2.95	2.00	0.95	0.21
			4.59	3.81	2.95	2.00
				5.30	4.59	3.81
					5.93	5.30
						6.51

- So, from this state, best choice is exercise option and "close mine"
  - This avoids larger losses <u>and</u>
  - … changes the NPV <u>as seen from that state</u>

Value seen from "5.93" state

- Seen from this state the value changes

From

5.93	5.30
	6.51

To

5.93	1.00
	1.00

- Expected PV from this state (that is, over the last year) then is loss over the last (6th) year, discounted over one year:

  = NPV [p (-1) + (1-p)(-1)] = - 0.89 (at 12%)

Note: fixed costs assumed to be unavoidable

Value seen from another state

- What if you were in best possible state at end of 5th year?

4.49	6.22
	2.92

- You would not close mine
- Expected PV for last year is discounted expectation over possible 6th year states:

  = NPV [p (6.22) + (1-p)(2.92)] = 4.82 (at 12%)
- Process can be repeated for each state…

Value seen for all states in 5th year

- To calculate the value of all states in the next to last year… we choose the better choice: "discounted value of maximum of keeping mine open or exercising option"

  = NPV[0.12, Max[EV(mine open), - 1]]
- Thus:

State	PV from 6th	Sum
4.49	4.82	9.30
1.50	2.00	3.50
0.95	0.30	1.25
2.95	0.89	3.84
4.59	0.89	5.48
5.93	0.89	6.83

Note rounding

To complete Analysis

- We need to repeat process by
- … estimating values for end of 4th year
- … then of 3rd, 2nd, 1st, until we get to start

0.76	2.42	6.00	8.66	9.93	9.30
	3.84	2.89	0.07	2.28	3.50
		4.71	3.84	2.89	1.25
			5.48	4.71	3.84
				6.19	5.48
					6.83

- In this case, project has an expected profit

Strategy Implied by Analysis

- Analysis determines the better choice at each node between exercising option or not
- Therefore, it provides strategy about when to exercise option
- In this case:

Strategy for exercise of option to close for example case

O	O	O	O	O	O
	CLOSE	CLOSE	O	O	O
		CLOSE	CLOSE	O	O
			CLOSE	CLOSE	CLOSE
				CLOSE	CLOSE
					CLOSE

Look ahead…

- The process we have just gone through, to evaluate value of option, is a simplified form of DYNAMIC PROGRAMMING
- Later presentation will formalize the general procedure
- DP is especially appropriate for finding optimum in Non-Convex feasible regions (as is that defined by lattice, see slide 20)

What is Value of Option?

- Value of the option is the increase in expected value due to flexibility
- In this example:

Base Case	with option	option value
$398,112	$763,158	$1,161,270

- The put option "on" the system is valuable – this 'insurance' against bad prices makes project attractive

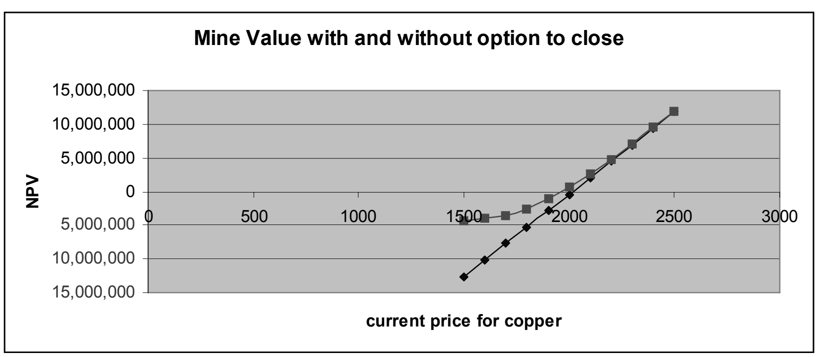
How does option value change?

- A data table shows variation of option value
- For example, with "Current Price of Copper"

	Base Case	with option	option value
	398,112	763,158	1,161,270
1500	12,632,806	4,446,161	8,186,644
1600	10,185,867	3,975,140	6,210,727
1700	7,738,928	3,504,119	4,234,809
1800	5,291,989	2,643,176	2,648,813
1900	2,845,050	981,639	1,863,412
2000	398,112	763,158	1,161,270
2100	2,048,827	2,704,462	655,635
2200	4,495,766	4,817,839	322,073
2300	6,942,705	7,127,023	184,318
2400	9,389,644	9,491,906	102,262
2500	11,836,582	11,887,531	50,948
2600	14,283,521	14,312,377	28,855
2700	16,730,460	16,746,595	16,135
2800	19,177,399	19,183,247	5,848
2900	21,624,338	21,628,324	3,986
3000	24,071,277	24,073,741	2,464

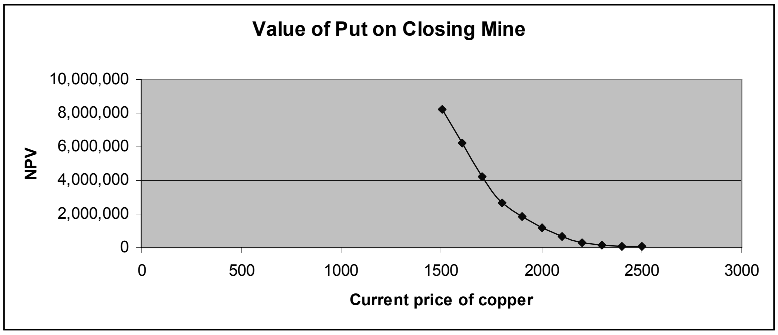
How "put" protects against losses

We can plot the sensitivity data to show how "put" option protects against losses



“Put” Insurance most valuable when risks greatest

As shown by plot of value of this “put”



Flexibility “in” the System

- Suppose that we design mine with extra vertical shaft, which enables increase in
  - annual production to 8000 tons
  - But then increases variable cost to $2400/ton
- This is a call option “in” the system
  - An “option” because it is “right, not obligation” to increase production
  - A “call” because it takes advantage of gains
  - “in” system, because it changes its technology
- What is the value of this option?

Valuation Process as before…

- Difference is in effect of exercising option
- Consider decision at a particular state

2.00	0.95	0.21	1.50	2.92	4.49	6.22
	2.95	2.00	0.95	0.21	1.50	2.92
		3.81	2.95	2.00	0.95	0.21
			4.59	3.81	2.95	2.00
				5.30	4.59	3.81
					5.93	5.30
						6.51

- Consider when prices highest (boxed cell)
  - Note: cells do not reflect cost of new shaft – we compare this to EV of option to see if worthwhile

At this high state…

- Revenues depend on whether operators exercise option
- If NO, then revenues as before:

4.49	6.22
	2.92

- If YES, then production and revenues increase, and pay extra. The net is

4.49	8.95
	3.67

- In this case, obvious that exercising option is better, and its net results go into lattice
- The process then continues as with "put"

Summary: 2 Main Ideas

- **Three elements combine in valuation of option**
  - Possible States of an Uncertainty
  - Probability this State may occur
  - Impact of States on Performance of System

- **Mechanics of process are like decision analysis calculation of value of information**
  - Difference due to recombinatorial nature of lattice
  - Need to focus clearly on effects of exercising option and, at each stage, choosing better of choices – to exercise or note
  - Process then repeats "from right to left"