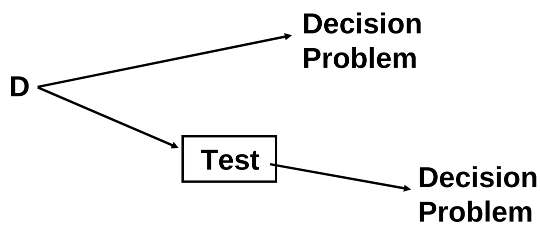
Information Collection - Key Strategy

- **Motivation**
  - To reduce uncertainty which makes us choose "second best" solutions as insurance
- **Concept**
  - Insert an information-gathering stage (e.g., a test) before decision problems, as a possibility



Operation of Test

New Information

↓

Revision of Prior Probabilities in Decision Problem

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New Expected Values in Decision Problem

EV (after test) $\geq$ EV (without test) Why?

- Because we can avoid bad choices and take advantage of good ones, in light of test results

- **Questions:**
  - Since test generally has a cost, is the test worthwhile?
    What is the value of information?
    Does it exceed the cost of the test?

Essential Concept

- Value of information is an expected value
- Expected Value of information
  = EV (after test) - EV (without test)

Concept application is complex!

- Expected Value of information
  $= \Sigma\, p_k\, [EV(D_k^*)] – EV\ (D^*)$

Where:

D* is optimal decision without test

D_k* are optimal decisions after test, based on k test results, TR_k , each of which revise probabilities from p_j to p_{jk}

p_k = probability, after test, of k^{th} observation

- Example: Test → Good / Medium / Poor
- Revise probabilities after each test result

… as an example

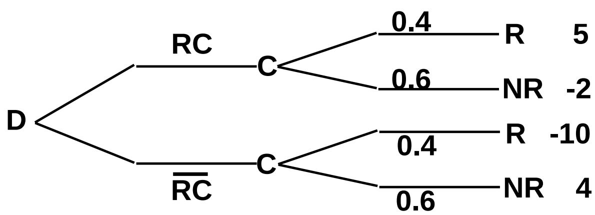
- Expected Value of information $= \Sigma\, p_k\, [EV(D_k^*)] - EV(D^*)$
- Current best design, D^*
- … has an expected value of EV (D^*)
- We run lab tests with possible outcomes, k (e.g, success, failure, …), each with prior, p_k
- Each test leads different best designs, D_k^*
- Each with a different expected value, $EV(D_k^*)$
- For a total value, post test, $\Sigma\, p_k\, [EV(D_k^*)]$

Expected Value of Perfect Information EVPI

- Perfect information is hypothetical – but simplifies!
- Use: Establishes upper bound on value of any test
- Concept: Imagine a "perfect" test which indicated exactly which Event, E_j, will occur (Cassandra)
  - By definition, this is "best" possible information
  - So, the "best" possible decisions can be made
  - and, the EV gain over the "no test" EV must be the maximum possible ; upper limit on the value of any test!

EVPI Example (1)

- Question: Should I wear a raincoat?
  RC – Raincoat $\overline{RC}$ - No Raincoat
- Two possible Uncertain Outcomes
  Rain: (p = 0.4)   No Rain: (p = 0.6)



- Remember that better choice is to take raincoat, EV = 0.8

EVPI Example (2)

- Perfect test

Why these probabilities? Because these are best estimates of results. Every time it rains, perfect test will say "rain"

- EVPI

EV (after test)	=	0.4(5) + 0.6(4) =	4.4
EVPI = 4.4 - 0.8 = 3.6			

Application of EVPI

- A major advantage: EVPI is simple to calculate
- Notice:
  - Prior probability (occurrence of uncertain event)
    MUST EQUAL
    probability (associated perfect test result)
  - For "perfect test", the posterior probabilities are either 1 or 0 (no doubt remains)
  - Optimal choice generally obvious, once we "know" what will happen
- Therefore, EVPI can generally be written directly
- No need to use Bayes' Theorem

Expected Value of Sample Information EVSI

- Sample information are results taken from an actual test
- Real Tests can improve estimate, some doubt generally remains
- Value of actual test not as good as hypothetical perfect test: $0 \leq \text{EVSI} \leq \text{EVPI}$
- Complex Calculations needed to account for persisting doubts…

EVSI Calculations required

- Obtain probabilities of each result TR_k , p_k
- For each test result TR_k
  - Revise prior probabilities p_j => p_{jk}
  - Calculate best decision D_k* (Note: this is a k- fold repetition of the original decision problem!!)
- Calculate EV (after test) = $\Sigma_k \; p_k (D_k^*)$
- Calculate EVSI as the difference between EV (after test) - EV (without test)
- A BIG JOB

EVSI Example

- Test consists of listening to forecasts
- Two possible test results
  - Rain predicted = RP
  - Rain not predicted = NRP
- Assume probability of a correct forecast = 0.7
  - p(RP/R) = p(NRP/NR) = 0.7
    says “rain” or “no rain” and that is what occurs
  - p(NRP/R) = p(RP/NR) = 0.3
    the prediction of “rain” or “no rain” is wrong

EVSI Example (1)

- **First calculation: probabilities of test results**

P(RP) = p(RP/R) p(R) + p(RP/NR) p(NR)
= (0.7) (0.4) + (0.3) (0.6) = 0.46

The prediction of "rain" occurs both from
correct forecasts when it is to rain AND
wrong forecasts when it does not rain

P(NRP) = 1.00 - 0.46 = 0.54
Since in this case it either rains or does not

EVSI Example (2)

- **Next: Posterior Probabilities**

P(R/RP) = p(R) (p(RP/R)/p(RP)) = 0.4(0.7/0.46) = 0.61
P(NR/NRP) = 0.6(0.7/0.54) = 0.78

Therefore,

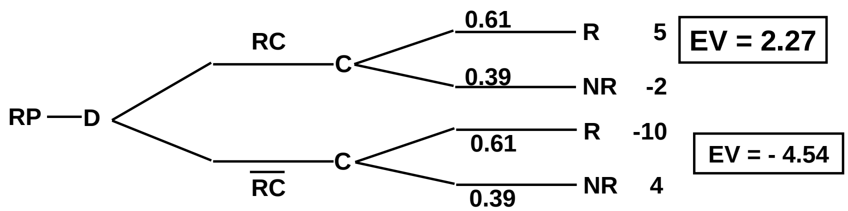
- **p(NR/RP) = 0.39 (<u>false positive</u> – says it will happen and it does not)**
- **p(R/RNP) = 0.22 (<u>false negative</u> – says it will not happen, yet it does)**

False Positive Example

- Prior Probability of Disease = 0.0001
- Accuracy of Test = P(Disease if test predicts) = P (D/DP). Assume = 0.001 and = P(ND/DNP)
- What is P(Disease/after test)?
- What is probability that test will report disease?
- Almost all the times you have it: (~1) (0.0001)
- In Error (0.001) (~1) ~ 0.001
- In this case, <u>false positive</u> ~ 10x true positive

EVSI Example (3)

- Best decisions conditional upon test results
- First, <u>if rain predicted</u>:

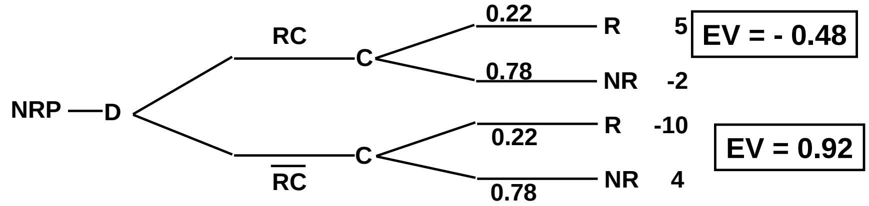


EV (RC) = (0.61) (5) + (0.39) (-2) = 2.27

EV ($\overline{RC}$) = (0.61) (-10) + (0.39) (4) = - 4.54

EVSI Example (4)

- **Best decisions <u>if No Rain Predicted</u>**



EV (RC) = (0.22) (5) + (0.78) (-2) = -0.48

EV ($\overline{RC}$) = (0.22) (-10) + (0.78) (4) = 0.92

EVSI Example (5)

- **EV (after test)**

  = p(rain predicted) (EV(strategy/RP))

  + P(no rain predicted) (EV(strategy/NRP))

  = 0.46 (2.27) + 0.54 (0.92) = 1.54

- **EVSI = 1.54 - 0.8 = 0.74**
- **EVSI < EVPI = 3.6 as indicated earlier**

Practical Example: Is a Test Worthwhile? (1)

- **If value is Linear (i.e., probabilistic expectations correctly represent value of uncertain outcomes)**
  - Calculate EVPI
  - If EVPI < cost of test → Reject test
  - Pragmatic rule of thumb
    If cost > 50% EVPI → Reject test
    (Real test are not close to perfect)
  - Calculate EVSI
  - EVSI < cost of test → Reject test
  - Otherwise, accept test

Is Test Worthwhile? (2)

- **If Value Non-Linear (i.e., EV of outcomes does NOT reflect attitudes about uncertainty)**
- **Theoretically, cost of test should be deducted from EACH outcome that follows a test**
  - If cost of test is known
    A) Deduct costs
    B) Calculate EVPI and EVSI (cost deducted)
    C) Proceed as for linear EXCEPT
       Question is if EVPI(cd) or EVSI(cd) > 0?
  - If cost of test is not known
    A) Use iterative, approximate pragmatic approach
    B) Focus first on EVPI
    C) Use this to estimate maximum cost of a test