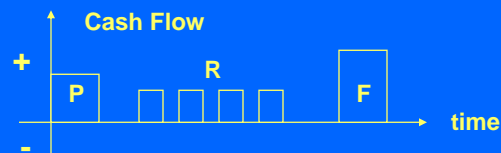


Present Value, Discounted Cash Flow. Engineering Economy

- **Objective:**
 - To provide economic comparison of benefits and costs that occur over time
- **Assumptions:**
 - All Benefits, Costs measured in money
 - Single point of view



Issue - Value over time

- **Money now has a different value than the same amount at a different date**
- **Comparable to – but not equal to – interest rate**
- **Proper name: Discount Rate, r**
 - because future benefits/costs are reduced...
(that is, “discounted”) to compare with present

Formulas for N Periods

- **Single amounts**

a) Future Amount = $P(1+r)^N = P(\text{caf})$
caf = Compound Amount Factor

b) Present Amount = F/caf
 $1/\text{caf}$ = Present Worth Factor

- **Finite Series**

c) $F = \sum_i R(1+r)^i = R[(1+r)^N - 1]/r$

d) $R = P(\text{crf}) = [P*r(1+r)^N] / [(1+r)^N - 1]$
crf = Capital Recovery Factor

Formulas for N Periods (cont')

- **Infinite Series**

$$1 \ll (1+r)^N \Rightarrow (1+r)^N / [(1+r)^N - 1] \Rightarrow 1$$

$$\Rightarrow \text{crf} = r$$

- **Small Periods**

$$(1+r)^N \Rightarrow e^{rN}$$

Present Value Analysis

- **Present Value Analysis puts all future amounts on a common basis, typically “the present”**
 - It may be some other reference year that is convenient, such as year of proposed investment
- **“Net” Present Value is the total of the present values of all future amounts, typically:
Revenues – Costs = Net Amount**
- **Referred to as “NPV”**

Example Application of Present Value Analysis

All by spreadsheets!

Example:

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Investment	15			3			5			
Net Income		2	3	4	5	5	3	4	5	6
Cash Flow	-15	2	3	1	5	5	-2	4	5	6
NPV at 12%	\$0.79									

Formula: NPV(12%, B9:K9)

Effect of Different Discount Rates

- **Higher Discount rates =>**
 - smaller value of future benefits
 - discourages projects with long pay back periods
 - project advocates try to minimize discount rate
 - Examples: Massive Dams ; Airports; Aircraft projects
- **Argument over Discount rates**
 - Often very difficult politically
 - Not hard from technical perspective
 - Generally higher than politicians want,
 - for example, US Government base case before 2000 was 7%, but under current administration it is ~5% (see later slides)
 - MORE DISCUSSION NEXT WEEK

US Govt base position on Discount rate (OMB Circular A-94,1992,revised yearly)

1. Base-Case Analysis.

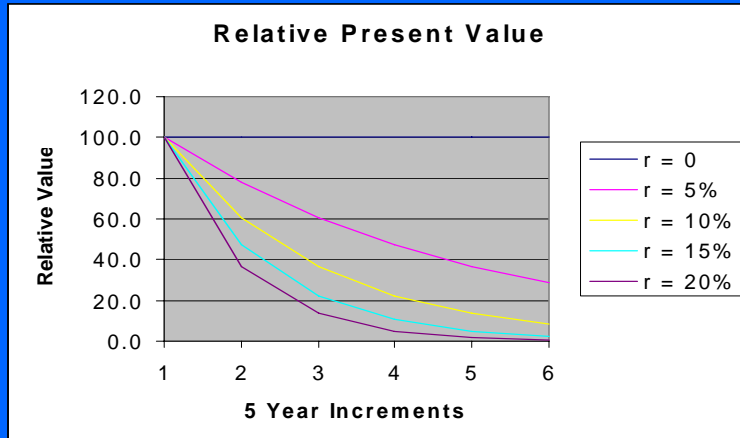
Constant-dollar benefit-cost analyses of proposed investments and regulations should report net present value and other outcomes determined using a real discount rate of 7 percent. This rate approximates the marginal pretax rate of return on an average investment in the private sector in recent years. Significant changes in this rate will be reflected in future updates of this Circular.

2. Other Discount Rates.

Analyses should show the sensitivity of the discounted net present value and other outcomes to variations in the discount rate. The importance of these alternative calculations will depend on the specific economic characteristics of the program under analysis. For example, in analyzing a regulatory proposal whose main cost is to reduce business investment, net present value should also be calculated using a higher discount rate than 7 percent.

<http://www.whitehouse.gov/omb/circulars/a094/a094.html> (accessed pre 2000)

Graphical view of Effect of Different Discount Rates and Lengths of Time



Discount Rate Approximation

- To appreciate effect of discounting:
“Rule of 72” or “Rule of 70”
 $e^{rN} = 2.0$ when $rN = 0.72$ (actually = 0.693)
- Therefore, present amount doubles when future amount halves
 $rN = 72$ with r expressed in percent
- Examples
 - When would \$1000 invested at 10% double?
 - What is, at 9%, the value of \$1000 in 8 years?

Effect of Different Time Horizons

- **Longer Periods of Benefits**
 - Increase Present Values
 - Increment depends on discount rate
- **What length of time matters?**
 - For US Government Rate, not much over 30 years
 - For Rates commonly used in business (15 to 20%), anything over 20 years has little value
 - Exception: If future benefits grow exponentially, they may compensate for discounting of future revenues

Current US Government position on Discount rate (OMB Circular A-94)

Provides 2 rates:

Nominal represents future purchasing power; reflects inflation

Real represents constant-dollar; assumes no inflation

=> Difference implies assumed rate of inflation

These rates change yearly (as of last several years)

These rates differ according to period – lower rates for shorter periods

**Note: assumed rate of inflation differs by period
(See next slides)**

Discount rates by OMB Circ. A-94, Appendix C [Ref: www.whitehouse.gov/omb/circulars/a094/a094_appx-c.html]

Nominal Discount Rates. A forecast of nominal or market interest rates for 2007 based on the economic assumptions from the 2008 Budget are presented below. These nominal rates are to be used for discounting nominal flows, which are often encountered in lease-purchase analysis.

Nominal Interest Rates on Treasury Notes and Bonds of Specified Maturities (in percent)

3-Year	5-Year	7-Year	10-Year	20-Year	30-Year
4.9	4.9	4.9	5.0	5.1	5.1

Real Discount Rates. A forecast of real interest rates from which the inflation premium has been removed and based on the economic assumptions from the 2008 Budget is presented below. These real rates are to be used for discounting real (constant-dollar) flows, as is often required in cost-effectiveness analysis.

Real Interest Rates on Treasury Notes and Bonds of Specified Maturities (in percent)

3-Year	5-Year	7-Year	10-Year	20-Year	30-Year
2.5	2.6	2.7	2.8	3.0	3.0

However...

- Guidelines appear precise (2 significant figures)
- But they are highly variable from year to year (see next slide), so analysis done in year N can be significantly off in year N+1
- Would seem to be political, as evidenced by comparing rates from last 6 years with those given before 2000 (see slide 8)

Previous Discount rates by OMB. A-94

For Year	Nominal Discount Rates for years into future					
	3	5	7	10	20	30
2006	4.7	4.8	4.9	5.0	5.3	5.2
2005	3.7	4.1	4.4	4.6		5.2
2003	3.1	3.6	3.9	4.2		5.1
2002	4.1	4.5	4.8	5.1		5.8

For Year	Real Discount Rates for years into future					
	3	5	7	10	20	30
2006	2.5	2.6	2.7	2.8	3.0	3.0
2005	1.7	2.0	2.3	2.5		3.1
2003	1.6	1.9	2.2	2.5		3.2
2002	2.1	2.8	3.0	3.1		3.9

Summary

- Formulas Simple
- Especially by Spreadsheets
- Discount rate is key issue
- High rates recommended (but see later presentations)
- Longer term benefits not large