Capacity Expansion

Objectives of Presentation:

- Explore the theory governing economics of capacity expansion (builds on assignment)
- Identify the fundamental tradeoffs
- Illustrate the sensitivity of the results for optimal policy – in deterministic conditions
- Provide a basis for understanding how results will change when future is uncertain

The Issue

- Given a system with known capacity
- We anticipate growth in future years

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<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
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<tbody>
<tr>
<td>Demand increase</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>…</td>
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- The question is: What size of capacity addition minimizes present costs?
Consider “infinite” horizon

- If growth is steady (e.g.: constant annual amount or percent),
- Does optimal policy change over time?

- Policy should not change -- Best policy now should be best at any other comparable time
- If we need addition now, and find best to add for N years, then best policy for next time we need capacity will also be an N year addition
- Best policy is a repetitive cycle

Cost assumptions

- Cost of Capacity additions is:
  - Cost = K (size of addition) \( a \)
  - If \( a < 1.0 \) we have economies of scale

- Cost of a policy of adding for N years, \( C(N) \), of growth annual growth “\( \Delta \)” is:
  - Initial investment = K (N \( \Delta \))\( a \)
  - Plus Present Values of stream of future additions. Value in N years is \( C(N) \), which we discount:
    \[
    C(N) = K (N \Delta)^a + e^{-rN} C(N)
    \]
If Constant Returns to Scale

- This means exponent $a = 1$
- Thus for example

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<tr>
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<td>25</td>
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<tr>
<td>Capacity Addition</td>
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<tr>
<td>Proportional Cost</td>
<td>15</td>
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<tr>
<td>Present Value at 10% = C(t=0) + C/r (for period)</td>
<td>$15 + 15(0.33)$</td>
<td>15</td>
<td>15</td>
<td>15</td>
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- Present Value for cyclical payments in Excel is:
  \[= \text{PV(DR over cycle beyond year 0, number of cycles, amount/cycle)}\]

- How does present cost vary over longer cycles?

Optimal policy in linear case

- The general formula for the present value of a series of infinite constant amounts is:
  \[\text{PV} = (\text{Constant Amount})\left(\frac{1}{1 + \frac{\text{DR over period}}{\text{DR over period}}}\right)\]

- In linear case:
  - Constant Amount = $K \cdot (N \Delta)$
  - DR over period = $N \cdot (\text{annual DR})$
  \[\text{PV} = K(N \Delta) \left[1 + \frac{1}{N \cdot (\text{annual DR})}\right] = K(\Delta) \left[N + 1 \cdot (\text{annual DR})\right]\]
- In this case, contributions of future cycles is irrelevant. Only thing that counts is first cost, which \[= f (N \Delta)\] that we minimize.
Consider diseconomies of scale

- In this case, it is always more expensive per unit of capacity to build bigger – that is meaning of diseconomies of scale
- Thus, incentive to build smaller
- Any countervailing force?
  - No!
- In this case, shortest possible cycle times minimize present costs

... and economies of scale?

- Economies of scale mean that bigger plants => cheaper per unit of capacity
- Thus, incentive to build larger
- Any countervailing force?
  - Yes!
- Economies of scale are a power function so advantage of larger units decreases, while they cost more now
- Best policy not obvious
Manne’s analysis

- Alan Manne developed an analytic solution for the case of constant linear growth

- Reference:

Here’s the picture (from Manne)
Design implications

- For industries with Economies of Scale, such as electric power, chemical processes
- For conditions assumed (constant steady growth forever) …
  - Small plants (for small N) are uneconomical
  - Optimum size in range of between 5 and 10 years
  - Corresponding to maybe 30 to 50% expansion
  - Results not especially sensitive to higher N
  - Which is good because forecasts not accurate.

What if assumptions wrong? (1)

- Growth assumed “for ever” …
- Is this always so?

- Not always the case:
  - Fashion or technology changes
  - Examples –
    - Analog film -- once digital cameras came in
    - Land lines – once everyone had cell phones
Optimal policy if growth stops

- What is the risk?
- Building capacity that is not needed

- What can designer do about it?
- Build smaller, so less reliance on future

- What is the cost of this policy?
- Tradeoff: extra cost/unit due to small plants, vs. saving by not paying for extra capacity

What if assumptions wrong? (2)

- Growth assumed steady… (no variability)
- Is this always so?

- Generally not correct.
  - Major economic cycles are common
  - Examples:
    - Airline industry
    - Consumer goods of all sorts
    - Industrial products…
Optimal policy if growth varies

- What is the risk?
- Building capacity not needed for long time

- What can designer do about it?
- Build smaller, more agile response to future

- What is the cost of this policy?
- Tradeoff: extra cost/unit due to small plants, vs. saving by not paying for extra

Example: Power in S. Africa

- Eskom, power generator for South Africa, adopted Manne’s model in mid-1970s
- They committed to units of 6 x 600 MW
- This led them to over 40% overcapacity within 15 years – an economic disaster!

Why did oversupply persist?

- Build up of oversupply was not sudden, lasted over a decade
- Why did system managers let this happen?

  - Not clear. Some likely explanations:
    - Managers committed to plan – thus it was too shameful to admit error of fixed plan
    - They had locked themselves in contractually – no possibility to cancel or defer implementation

- In any case, system was inflexible
What could have been done?

- They would have been better off (less unnecessary or premature construction) if they had:
  - not committed themselves to a fixed plan, but had signaled possibility of change
  - contractual flexibility to cancel orders for turbines, defer construction (they would have had to pay for this, of course)
  - Planned for smaller increments

- In short, they should have been flexible

General Cap. Expan. Problem

- How should system designers “grow” the system over time?

  - Main tensions:
    - For early build – if economies of scale
    - For delays –
      - defer costs, lower present values
      - resolution of uncertainties

  - Need for flexible approach
Summary

- Capacity Expansion Issue is a central problem for System Design
- Deterministic analysis offers insights into major tradeoffs between advantages of economies of scale and of deferral
- In general, however, the uncertainties must be considered
- A flexible approach is needed
  - How much? In what form?
- This is topic for rest of class

Appendix

- Details on Calculations and results from Alan Manne
- These replicate what you found in homework exercise (optimal capacity expansion)
Applications of M’s Analysis

- Still widely used
- In industries that have steady growth
- In particular in Electric power
- See for example


Details of Manne’s analysis (1)

- Cost of Stream of replacement cycles is:
  \[ C(N) = K (N \Delta)^a + e^{-rN} C(N) \]

- Thus: \[ C(N) = K (N \Delta)^a / [1 - e^{-rN}] \]

- This can be optimized with respect to \( N \), decision variable open to system designer
Details of Manne’s analysis (2)

- It’s convenient to take logs of both sides
  \[
  \log [C(N)] = \log [K (N \Delta)^a] - \log [1 - e^{-rN}]
  = \log [N^a] + \log [K (\Delta)^a] - \log [1 - e^{-rN}]
  \]
  \[
  \frac{d(\log [C(N)])}{dN} = \frac{a}{N} - \frac{r}{[e^{-rN} - 1]} = 0
  \]
- So: \( a = \frac{rN^*}{[e^{-rN^*} - 1]} \)
- Optimal cycle time determined by \((a, r)\) -- for assumed conditions

Key results from Manne

- There is an optimal cycle time…
- It depends on
  - Economies of scale: lower “a” \(\Rightarrow\) longer cycle
  - Discount rate: higher DR \(\Rightarrow\) shorter cycle

- It does not depend on growth rate!
- Higher growth rate \(\Rightarrow\) larger units
- However, this is driven by cycle time
  Capacity addition = (cycle time)(growth rate)
Graphical View of Results

- Manne’s present costs versus cycle time
  - steep for small cycle times
  - quite flat at bottom and for larger cycle times

- See following figure…

- What are implications?

Sensitivity of results (Manne)

- Total Cost vertically
- Cycle Time, $N$, horizontally
- Optimum is flat, around 6.75 years for $a = 0.7$ and $r = 0.1 = 10\%$