

# Capacity Expansion

## Objectives of Presentation:

- Explore the theory governing economics of capacity expansion (builds on assignment)
- Identify the fundamental tradeoffs
- Illustrate the sensitivity of the results for optimal policy – in deterministic conditions
- Provide a basis for understanding how results will change when future is uncertain

# The Issue

- Given a system with known capacity
- We anticipate growth in future years

Year	0	1	2	3	4	5	6	...	N
Demand increase	0	5	10	15	20	25	30	...	5N

- The question is:  
What size of capacity addition  
minimizes present costs ?

## Consider “infinite” horizon

- If growth is steady (e.g.: constant annual amount or percent),
- Does optimal policy change over time?
  
- Policy should not change -- Best policy now should be best at any other comparable time
- If we need addition now, and find best to add for N years, then best policy for next time we need capacity will also be an N year addition
- Best policy is a repetitive cycle

## Cost assumptions

- Cost of Capacity additions is:
  - Cost = K (size of addition)<sup>a</sup>
  - If  $a < 1.0$  we have economies of scale
  
- Cost of a policy of adding for N years, C(N), of growth annual growth “ $\Delta$ ” is:
  - Initial investment =  $K (N \Delta)^a$
  - Plus Present Values of stream of future additions. Value in N years is C(N), which we discount:

$$C(N) = K (N \Delta)^a + e^{-rN} C(N)$$

## If Constant Returns to Scale

- This means exponent  $a = 1$
- Thus for example

Year	0	1	2	3	4	5	6	...	N
Demand Increase	0	5	10	15	20	25	30	...	5N
Capacity Addition	15			15			15	...	
Proportional Cost	15			15			15	...	
Present Value at 10% = $C(t=0) + C/r$ (for period)	15 + 15/(0.33)								
	= 60								

- Present Value for cyclical payments in Excel is:  
= PV(DR over cycle beyond year 0, number of cycles, amount/cycle)
- How does present cost vary over longer cycles?

## Optimal policy in linear case

- The general formula for the present value of a series of infinite constant amounts is:  
 $PV = (\text{Constant Amount}) [1 + 1 / (\text{DR over period})]$
- In linear case:
  - > Constant Amount =  $K (N \Delta)$
  - > DR over period =  $N$  (annual DR)
  - >  $PV = K(N \Delta) [1 + 1 / N \text{ (annual DR)}] = K(\Delta) [N + 1 / \text{(annual DR)}]$
- In this case, contributions of future cycles is irrelevant. Only thing that counts is first cost, which  $[ = f (N \Delta) ]$  that we minimize.

## **Consider diseconomies of scale**

- In this case, it is always more expensive per unit of capacity to build bigger – that is meaning of diseconomies of scale
- Thus, incentive to build smaller
- Any countervailing force?
  
- No !
- In this case, shortest possible cycle times minimize present costs

## **... and economies of scale?**

- Economies of scale mean that bigger plants => cheaper per unit of capacity
- Thus, incentive to build larger
- Any countervailing force?
  
- Yes!
- Economies of scale are a power function so advantage of larger units decreases, while they cost more now
- Best policy not obvious

## Manne's analysis

- Alan Manne developed an analytic solution for the case of constant linear growth
- Reference:  
**Manne, A. (1967) Investments for Capacity Expansion: Size, Location and Time-phasing, MIT Press, Cambridge, MA**

## Here's the picture (from Manne)

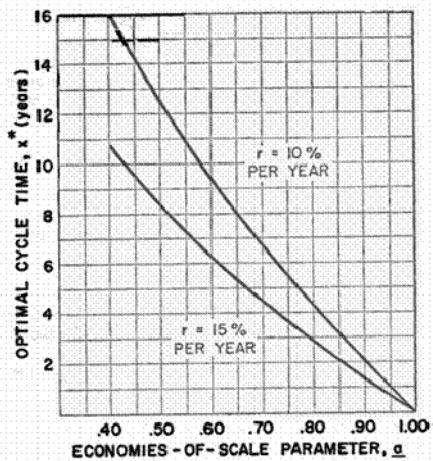


Figure 2.4.—Cross-Plot for Optimal Cycle Time

## Design implications

- For industries with Economies of Scale, such as electric power, chemical processes
- For conditions assumed (constant steady growth forever) ...
  - Small plants (for small N) are uneconomical
  - Optimum size in range of between 5 and 10 years
  - Corresponding to maybe 30 to 50% expansion
  - Results not especially sensitive to higher N
  - Which is good because forecasts not accurate.

## What if assumptions wrong? (1)

- Growth assumed “for ever” ...
- Is this always so?
- Not always the case:
  - Fashion or technology changes
  - Examples –
    - ✱ Analog film -- once digital cameras came in
    - ✱ Land lines – once everyone had cell phones

## Optimal policy if growth stops

- What is the risk?
  - Building capacity that is not needed
- What can designer do about it?
  - Build smaller, so less reliance on future
- What is the cost of this policy?
  - Tradeoff: extra cost/unit due to small plants, vs. saving by not paying for extra capacity

## What if assumptions wrong? (2)

- Growth assumed steady...(no variability)
- Is this always so?
- Generally not correct.
  - Major economic cycles are common
  - Examples:
    - \* Airline industry
    - \* Consumer goods of all sorts
    - \* Industrial products....

## Optimal policy if growth varies

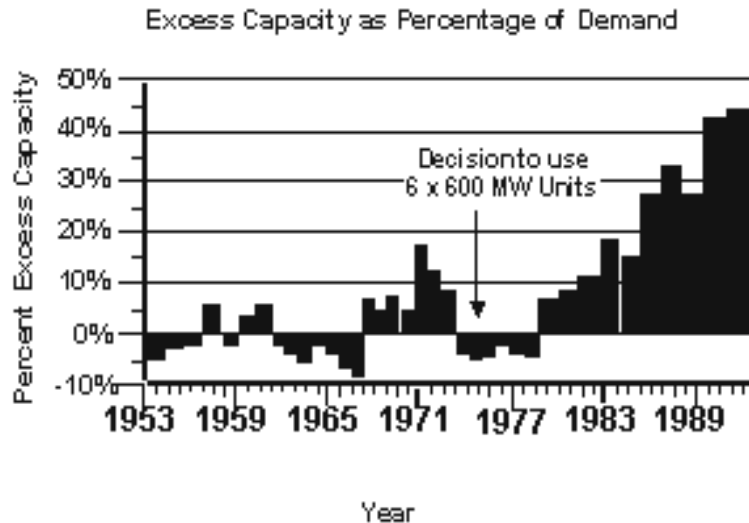
- What is the risk?
- Building capacity not needed for long time
  
- What can designer do about it?
- Build smaller, more agile response to future
  
- What is the cost of this policy?
- Tradeoff: extra cost/unit due to small plants, vs. saving by not paying for extra

## Example: Power in S. Africa

- Eskom, power generator for South Africa, adopted Manne's model in mid-1970s
- They committed to units of 6 x 600 MW
- This led them to over 40% overcapacity within 15 years – an economic disaster!
  
- Aberdein, D. (1994) "Incorporating Risk into Power Station Investment Decisions in South Africa, S.M. Thesis, MIT, Cambridge, MA.



## Expansion of Established Industry Power Station Investment in South Africa



### Why did oversupply persist?

- Build up of oversupply was not sudden, lasted over a decade
- Why did system managers let this happen?
- Not clear. Some likely explanations:
  - Managers committed to plan – thus it was too shameful to admit error of fixed plan
  - They had locked themselves in contractually – no possibility to cancel or defer implementation
- In any case, system was inflexible

## **What could have been done?**

- **They would have been better off (less unnecessary or premature construction) if they had:**
  - not committed themselves to a fixed plan, but had signaled possibility of change
  - contractual flexibility to cancel orders for turbines, defer construction (they would have had to pay for this, of course)
  - Planned for smaller increments
- **In short, they should have been flexible**

## **General Cap. Expan. Problem**

- **How should system designers “grow” the system over time?**
- **Main tensions:**
  - For early build – if economies of scale
  - For delays –
    - \* defer costs, lower present values
    - \* resolution of uncertainties
- **Need for flexible approach**

## Summary

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- **Capacity Expansion Issue is a central problem for System Design**
- **Deterministic analysis offers insights into major tradeoffs between advantages of economies of scale and of deferral**
- **In general, however, the uncertainties must be considered**
- **A flexible approach is needed**
  - **How much? In what form?**
- **This is topic for rest of class**

## Appendix

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- **Details on Calculations and results from Alan Manne**
- **These replicate what you found in homework exercise (optimal capacity expansion)**

## Applications of M's Analysis

- Still widely used
- In industries that have steady growth
- In particular in Electric power
- See for example
  
- Hreinsson, E. B. (2000) "Economies of Scale and Optimal Selection of Hydropower projects," Proceedings, Electric Deregulation and Restructuring of Power Technologies, pp. 284-289.

## Details of Manne's analysis (1)

- Cost of Stream of replacement cycles is:  
$$C(N) = K (N \Delta)^a + e^{-rN} C(N)$$
- Thus:  $C(N) = K (N \Delta)^a / [1 - e^{-rN}]$
- This can be optimized with respect to N, decision variable open to system designer

## Details of Manne's analysis (2)

- It's convenient to take logs of both sides
$$\log [C(N)] = \log [K (N \Delta)^a] - \log [1 - e^{-rN}]$$
$$= \log [N^a] + \log [K (\Delta)^a] - \log [1 - e^{-rN}]$$

$$d(\log [C(N)]) / dN = a/N - r / [e^{-rN} - 1] = 0$$

- So:  $a = rN^* / [e^{-rN^*} - 1]$
- Optimal cycle time determined by (a, r)  
-- for assumed conditions

## Key results from Manne

- There is an optimal cycle time...
- It depends on
  - Economies of scale: lower "a" → longer cycle
  - Discount rate: higher DR → shorter cycle
- It does not depend on growth rate!
- Higher growth rate → larger units
- However, this is driven by cycle time  
Capacity addition = (cycle time)(growth rate)

## Graphical View of Results

- Manne's present costs versus cycle time
  - > steep for small cycle times
  - > quite flat at bottom and for larger cycle times
- See following figure...
- What are implications?

## Sensitivity of results (Manne)

- Total Cost vertically
- Cycle Time,  $N$ , horizontally
- Optimum is flat, around 6.75 years for  $a = 0.7$  and  $r = 0.1 = 10\%$

