Path Dependency in Option Valuation

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Outline

- Context: Computational Efficiency
- Issue: Path Dependency impact on above
- Examples of Issue:
  - Communications Satellite Constellation
  - Generality of Cases
- A Resolution procedure
- Summary

Computational Efficiency

- Tree Structures showing paths of development become grow exponentially as stages increase
  - Example: Consider a decision tree with only 2 choices each with 2 outcomes at each stage; the number of paths: 4, 16, 64, 256… ➔ 4^N
- Lattices however grow linearly, because of recombination of nodes
  - Example: Paths for a binary lattice = 2, 3, 4… ➔ N
- Path independency is key to this advantage:
  The future state for any node in lattice does not depend on how you get there

Validity of Path Independence

- Plausible for Market Prices
  - If price of asset is now $30, it seems reasonable to assume that it does not have a "memory" that influences future price
  - Future price is independent of previous prices such as (30, 32, 30) or (30, 28, 30)
- Is this necessary?
  - No! It is a conclusion based on ideal that markets have full information and no biases
  - Reality might be different – future price might be higher if people think asset is "moving up"
- However, on balance seems reasonable

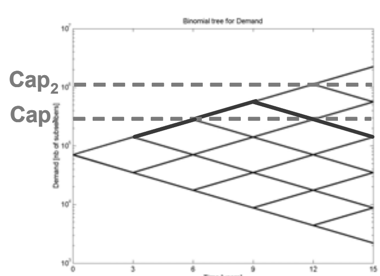
If Path Independence does not apply…

- Then recombinant feature of lattice does not exist…
- So that lattice becomes more dense…
- … and computational load increases
- Extent of change depends on degree to which path independence does not hold
- In any case, makes calculations more difficult

Example of Path Dependence

- Communications Satellite Case:
  - Decision rule: expand capacity when needed
  - Consequence: If "demand went up, then down," system may end up with different capacity compared to "down, then up"

See figure from de Weck. If demand follows red line, more satellites sent up, situation at end differs from "down, then up" paths to same point



Specific Consequences

- For example, for Communications Satellites:
- Expansion of capacity changes cost of system
- … Therefore, system profitability under future state of demand
- … Thus, analysis has to consider 2 different situations in future (i.e., at end of red line)
  - Larger system -- more costly, less profitable
  - Smaller system – less costly, more profitable
- Analysis "duplicated" from then forward

Generality of Path Dependence

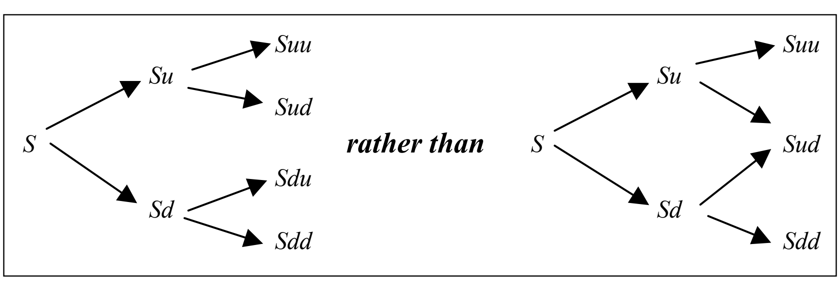
- Possibility of Path Dependence exists whenever designers of system develop in ways that affect future performance
- This situation is not covered in ordinary methods of "options analysis"
- A research area important to designers of engineering systems

Wang's procedure

- Stochastic Mixed-Integer Programming
- Stochastic… because it deals with variability..
- Mixed-Integer … meaning it is Linear Programming modified to take into account fact that some variables are not continuous (e.g., are "zero-one" in that they exist or not)
- Computationally heavy – but possible if number of stages is limited

Essence of Wang's Analysis

- Analysis presents problem as at Left



- Formulation as on next slide

Formulation – complex!

$$\text{Max} \quad \sum_s \sum_q p^q \sum_i \sum_t \beta^q_{Pi} P_{ist} [\sum_{j=1}^{i} R^q_{js} - (1-f) R^q_{is}] PV_i + \sum_s \sum_q \sum_i \sum_t p^q \beta^q_{PT} P_{ist} PVO_i R^q_{is}$$
$$- \sum_s \sum_q \sum_i p^q \{[\alpha_s(\overline{V_s}) + \delta_s(\overline{H_s})] R^q_{is} \cdot PVC_i\}$$

where

$$PV_i = \sum_{j=10(i-1)+1}^{10i} \frac{1}{(1+r)^j}$$

$$PV_o = \sum_{j=31}^{70} \frac{1}{(1+r)^j}$$

$$PVC_i = \frac{1}{(1+r)^{10(i-1)}}$$

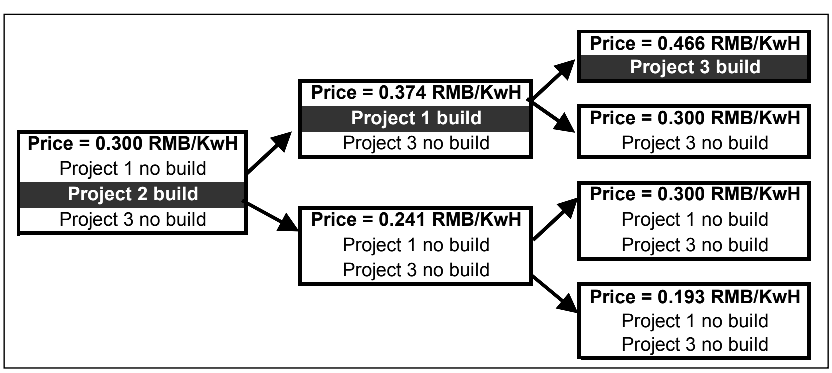
Likewise, the constraints differ because this formulation adds the real options constraints:

$$\sum_i R^q_{is} \le 1 \qquad \forall s, q$$

$$R^{q_1}_{is} = R^{q_2}_{is} \quad \forall (q_1, q_2) \text{ through node } k, \forall k \in \delta_i, \forall i = 1, \ldots, n$$

Example Results

- **Solution defines strategy conditional on levels of observable parameters**
  - **In this case, price of electricity from hydropower**



Summary

- Path Dependency makes Calculations difficult and expensive for many stages
- Path Dependency can be expected in the development of engineering systems
- Thus, efficient ways to solve "path dependency" problems are important
- …especially for real options "in" systems
- Stochastic Mixed-Integer Programming one way, others possible…

References

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- de Neufville, R. and Wang, T. (2004) "Building Real Options into Physical Systems with Stochastic Mixed Integer Programming," 8th Internatl Conf. on Real Options, Montreal, June http://www.realoptions.org/papers2004/