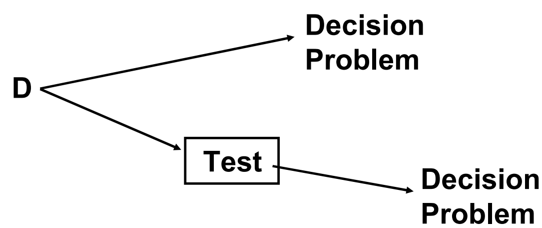
Information Collection- Key Strategy

- **Motivation**
  - To reduce uncertainty which makes us choose “second best” solutions as insurance
- **Concept**
  - Insert an information-gathering stage (e.g., a test) before decision problems, as a possibility



Operation of Test

New Information

↓

Revision of Prior Probabilities in Decision Problem

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New Expected Values in Decision Problem

EV (after test) $\geq$ EV (without test) Why?

- Because we can avoid bad choices and take advantage of good ones, in light of test results
- **Question:**
  - Since test generally has a cost, is the test worthwhile? What is the value of information? Does it exceed the cost of the test?

Essential Concept - it's complex!

- Value of information is an expected value
- Expected Value of information
  = EV (after test) - EV (without test)
  $= \sum_k p_k (D_k^*) - EV(D^*)$

Where D* is optimal decision without test and D_k^* are optimal decisions after test, based on k test results, TR_k, that revise probabilities from p_j to p_{jk}

p_k = probability, after test, of k^{th} observation

- Example

Test < Good / Medium / Poor

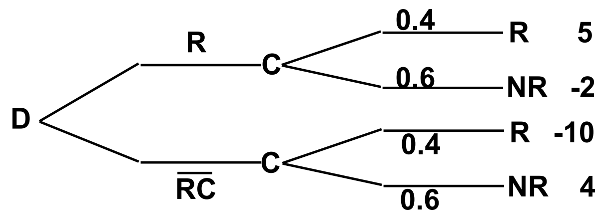
Revise probabilities after each test result

Expected Value of Perfect Information EVPI

- Perfect information is hypothetical – but simplifies!
- Use: Establishes upper bound on value of any test
- Concept: Imagine a "perfect" test which indicated exactly which Event, E_j, will occur
  - By definition, this is the "best" possible information
  - Therefore, the "best" possible decisions can be made
  - Therefore, the EV gain over the "no test" EV
    must be the maximum possible
    an upper limit on the value of any test!

EVPI Example (1)

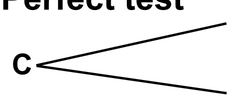
- **Question: Should I wear a raincoat?**
  RC - Raincoat; $\overline{RC}$ - No Raincoat
- **Two possible Uncertain Outcomes**
  (p = 0.4) or No Rain (p = 0.6)



- **Remember that better choice is to take raincoat, EV = 0.8**

EVPI Example (2)

- **Perfect test**



Says Rain p = 0.4 Take R/C 5

Says No Rain p = 0.6 No R/C 4

Why these probabilities? Because these are best estimates of results. Every time it rains, perfect test will say "rain"

- **EVPI**

EV (after test)	**=**	**0.4(5) + 0.6(4) =**	**4.4**
EVPI = 4.4 - 0.8 = 3.6			

Application of EVPI

- A major advantage: EVPI is simple to calculate
- Notice:
  - Prior probability of the occurrence of the uncertain event must be equal to the probability of observing the associated perfect test result
  - As a "perfect test", the posterior probabilities of the uncertain events are either 1 or 0
  - Optimal choice generally obvious, once we "know" what will happen
- Therefore, EVPI can generally be written directly
- No need to use Bayes' Theorem

Expected Value of Sample Information EVSI

- Sample information are results taken from an actual test $0 \leq \text{EVSI} \leq \text{EVPI}$
- Calculations required
  - Obtain probabilities of test results, p_k
  - Revise prior probabilities p_j for each test result TR_k => p_{jk}
  - Calculate best decision D_k^* for each test result TR_k (a k- fold repetition of the original decision problem)
  - Calculate EV (after test) = $\Sigma_k \; p_k (D_k^*)$
  - Calculate EVSI as the difference between EV (after test) - EV (without test)
- A BIG JOB

EVSI Example (1)

- **Test consists of listening to forecasts**
- **Two possible test results**
  - **Rain predicted = RP**
  - **Rain not predicted = NRP**
- **Assume probability of a correct forecast = 0.7**

  p(RP/R) = p(NRP/NR) = 0.7

  p(NRP/R) = p(RP/NR) = 0.3
- **First calculation: probabilities of test results**

  P(RP) = p(RP/R) p(R) + p(RP/NR) p(NR)

  = (0.7) (0.4) + (0.3) (0.6) = 0.46

  P(NRP) = 1.00 - 0.46 = 0.54

EVSI Example (2)

- **Next: Posterior Probabilities**

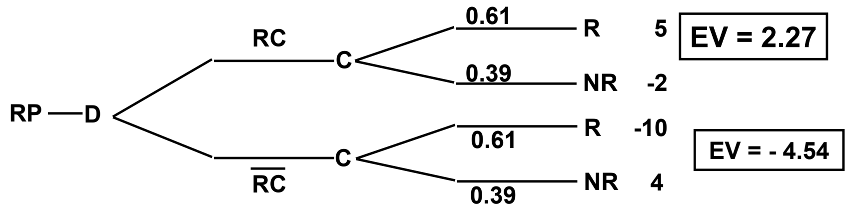
  P(R/RP) = p(R) (p(RP/R)/p(RP)) = 0.4(0.7/0.46) = 0.61

  P(NR/NRP) = 0.6(0.7/0.54) = 0.78

  Therefore,
- **p(NR/RP) = 0.39 (false positive – says it will happen and it does not)**
- **p(R/RNP) = 0.22 (false negative – says it will not happen, yet it does)**

EVSI Example (3)

- **Best decisions conditional upon test results**

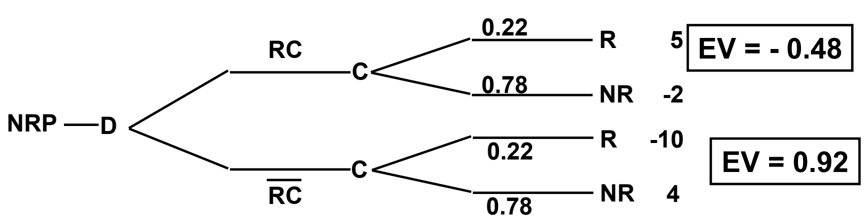


EV (RC) = (0.61) (5) + (0.39) (-2) = 2.27

EV ($\overline{RC}$) = (0.61) (-10) + (0.39) (4) = - 4.54

EVSI Example (4)

- **Best decisions conditional upon test results**



EV (RC) = (0.22) (5) + (0.78) (-2) = -0.48

EV ($\overline{RC}$) = (0.22) (-10) + (0.78) (4) = 0.92

EVSI Example (5)

- EV (after test)
  = p(rain predicted) (EV(strategy/RP))
  + P(no rain predicted) (EV(strategy/NRP))
  = 0.46 (2.27) + 0.54 (0.92) = 1.54
- EVSI = 1.54 - 0.8 = 0.74 < EVPI = 3.6

Practical Example: Is a Test Worthwhile? (1)

- If value is Linear (i.e., probabilistic expectations correctly represent value of uncertain outcomes)
  - Calculate EVPI
  - If EVPI < cost of test → Reject test
  - Pragmatic rule of thumb
    If cost > 50% EVPI → Reject test
    (Real test are not close to perfect)
  - Calculate EVSI
  - EVSI < cost of test → Reject test
  - Otherwise, accept test

Is Test Worthwhile? (2)

- If Value Non-Linear (i.e., EV of outcomes does NOT reflect attitudes about uncertainty)
- Theoretically, cost of test should be deducted from EACH outcome that follows a test
  - If cost of test is known
    A) Deduct costs
    B) Calculate EVPI and EVSI (cost deducted)
    C) Proceed as for linear EXCEPT
    Question is if EVPI(cd) or EVSI(cd) > 0?
  - If cost of test is not known
    A) Use iterative, approximate pragmatic approach
    B) Focus first on EVPI
    C) Use this to estimate maximum cost of a test