Adjusting discount rate for Uncertainty

- The Issue
- A simple approach: WACC
  - — Weighted average Cost of Capital
- A better approach: CAPM
  - — Capital Asset Pricing Model

Semantic Caution
Uses of the words “risk” and “uncertainty”

- Traditional Engineering assumes
  - — variability in outcomes leads to bad events
  - — equates uncertainty with downside, with “risk”
- But: variability may give upside opportunity
  - — so, we should generally think of “uncertainty”
  - — I will try to use this term whenever possible
- This presentation uses “risk” where the economic literature uses this term

Background: Aversion to "Risk"

- What is "risk aversion"?
- People prefer projects with less variability in return on investment
- Thus: people require some premium (extra payment) before they will accept projects with more uncertainty
- The result: people will want to adjust discount rate for uncertainty
- See examples…

Example

- A Simple Game:
  - I Am Ready to Give Away $1 On Coin Toss
  - If Heads, I Give Away; If Tails, I Keep Money
  - Probability of Heads = 50%
    Expected Value = $ 0.50
  - How Much Would You, Individually, Pay Me For The Opportunity To Play This Game?

Slightly Different Example

- A Repeat of Simple Game:
  - I Am Ready to Give Away $10 On Coin Toss
  - If Heads, I Give Away; If Tails, I Keep Money
  - Probability of Heads = 50%
    Expected Value = $ 5
  - How Much Would You, Individually, Pay Me For The Opportunity To Play This Game?

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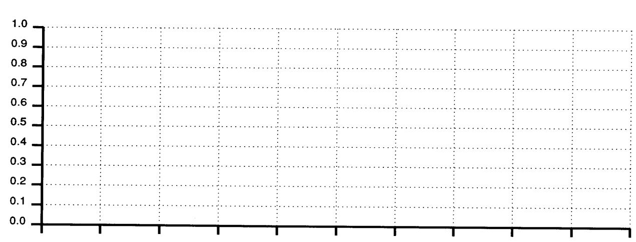
Interpretation of Example

- Averages Not The Basis For Most People's Choice
- People Decide on the Basis of "Real Value" ≡ Utility
- They are "Risk Averse" , their Utility Typically Is Non-Linear



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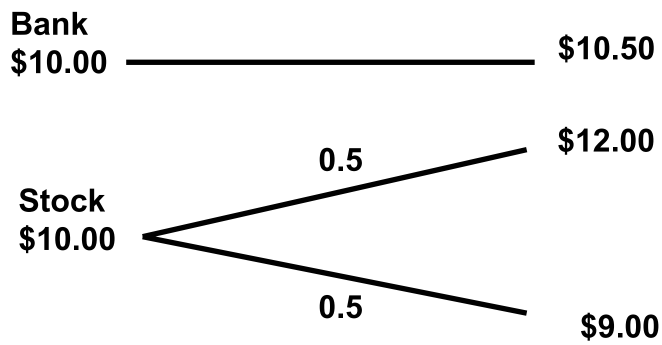
Consider this example...

- Consider two investments of $1000
  - Savings account with annual yield of 5%
  - Stock with a 50:50 chance of $1200 or $900 in a year



Investors Prefer Less Uncertainty

- Expected returns are identical:
  - Savings account = 5%
  - Stock = {[0.5*(1200 + 900) -1000] / 1000 }* 100% = 5%

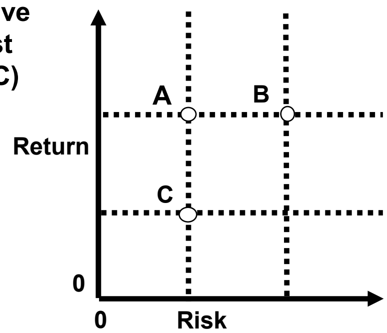
- Which would you prefer?

- In general, for same return, investors prefer project with more reliable, less uncertain returns

- What if stock had a 75% chance of selling for $1200? At some higher return, we prefer uncertain project

General Perspective on Risk vs Return

- Two key observations regarding preferences
- Non-satisfaction
  - For a given level of risk, the preferred alternative is one with the highest expected return (A > C)
- Risk Aversion
  - For a given level of return, the preferred alternative is one with the lowest level of risk (A > B)



Adjusting discount rate for Uncertainty -- simple approach

- Weighted Average Cost of Capital (WACC)
- Recall: WACC represents average return
  = R for equity (Equity %) + R on Bonds (Bond %)
- Returns on Equity and Bonds depend on "risk" of company. Established company generally more certain than start-up
- Thus: WACC represents risk of company

When is WACC good adjustment for uncertainty?

- WACC represents average for company
- … So, it may be right for average projects

- What adjustment right for unique projects?
- More generally, how do we define discount rates for projects in uncertain world?

- Note: Logic is that since projects uncertainties differ, so should their discount rates. A company thus might use several!

Adjusting discount rate for Uncertainty a better approach

- The Capital Asset Pricing Model (CAPM)
  - — Assumptions about investor attitudes
  - — Components of Uncertainty
  - — Principle of diversification
  - — Beta – a formal measure of "risk"
  - — CAPM relation between return and "risk"
  - — Expected return from unique projects
- Use of CAPM for project evaluation

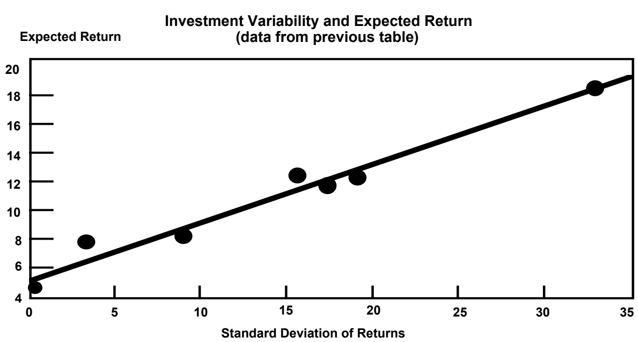
Some Observations on how returns vary with uncertainty

- "Risk-free" rate defined as return if no variability
- Investments with greater variability are riskier
- variability and expected return are correlated
- Suggestive data from a few years ago:

Security	Expected Return %	Variability: Standard Deviation of Expected Returns (%)
Risk free	5	0
U.S Treasuries	7.7	3.3
Fixed Income	9.0	9.0
Domestic Equity	12.7	18.5
International Equity	12.9	19.4
Real Estate	12.9	16.9
Venture Capital	18.6	33.0

Greater Variability => Greater Expected Return

- An upward trend



A Note on "risk-free" rate

- In one sense the "risk-free" rate is theoretical
  - what investment is entirely free of risk?
  - Note: you may be sure of getting money back, but may have lost due to inflation…
- In options analysis, "risk-free" rate needs a number
  - this is taken to be rate of US Government bonds
  - on grounds that these are safest investments (do not ask me to defend this view)
  - this rate depends on life of the bond, that is, the time to maturity (such as 6 months, 10 years…)

Components of Uncertainty

- Useful to recognize 2 types of uncertainties
- Using standard terms:
- Market Risk (systematic, non-diversifiable)
  - Investments tend to fluctuate with outside markets
  - Declines in the stock market generally affect all stocks
- Unique or Project Risk (idiosyncratic, diversifiable)
  - Individual characteristics of investments affect return
  - An investment might be better or worse than overall market trends, because of its special characteristics
- What compensation should investors demand for each type?

Diversification

- A collection of projects (a portfolio) 'diversifies' the variability in return (has different ones)
- It reduces Unique Risks
- Why is this?
- Because ups in one project counterbalance downs in others thus lowering variability of portfolio

Role of Diversification

- Consider this example of two stocks:
  — A: Expected return = 20%,
    Standard Deviation of Expected Returns = 20%
  — B: Expected Return = 20%
    Standard Deviation of Expected Returns = 20%
- If portfolio has equal amounts of A and B
  — Expected return = 0.5*20% + 0.5*20% = 20%
  — What is Standard Deviation?
- In general, standard deviation of return on portfolio is NOT average of that of individual stocks!

Standard Deviation for a Portfolio

- Portfolio standard deviation is not a weighted average

- Portfolio standard deviation

  $\sigma_p = \sqrt{\Sigma_i \Sigma_j x_i x_j \sigma_I \sigma_j \rho_{ij}}$

  for a portfolio of N investments, with i, j = 1 to N

  x_i , x_j = Value fraction of portfolio represented by investments i and j

  σ_I , σ_j = Standard deviation of) investments i and j

  ρ_{ij} = Correlation between investments i and j

  ρ_{jj} = 1.0

Standard Deviation of 2 Stock Portfolio

- Invest equal amounts in two stocks
  - For both A & B: Expected Return = 20%, Standard Deviation = 20%

$\sigma_p = \sqrt{(0.5)(0.5)(0.2)(0.2)(1) + (0.5)(0.5)(0.2)(0.2)(1) + (2)(0.5)(0.5)(0.2)(0.2)\rho_{ab}}$

- Portfolio standard deviation depends on correlation of A, B (ρ_{ij})

Correlation Between A & B	Portfolio Standard Deviation	Portfolio Expected Return
1	20.0%	20%
0.5	17.3%	20%
0	14.1	20%
-1	0.0%	20%

Conclusions from Example

- Most investments not perfectly correlated (correlation, $\rho_{ij} < 1$)
- Holding portfolio reduces standard deviation of value of portfolio, thus reduces "risk"
- With negative correlation, can eliminate all "risk"

Generalization for Many Stocks

- Formula for standard deviation σ_p of portfolio

$$\sigma_p = \sqrt{\Sigma_i \Sigma_j x_i x_j \sigma_i \sigma_j \rho_{ij}} = \sqrt{\text{portfolio variance}}$$

- For a portfolio of N stocks in equal proportions ($x_i = x_j = 1/N$)
  — N weighted variance terms, $i = j \rightarrow \sigma^2_i$
  — (N^2-N) weighted cov. terms, $i, j \rightarrow \sigma_i \sigma_j \rho_{ij}$
- Var(P) = N*(1/N)^2* Average Variance + (N^2-N)* (1/N)^2*Average Covariance
- Var(P) = (1/N)*Av. Variance + [1-(1/N)*] Av. Covariance

Implications of diverse portfolio

$\sigma_p = \sqrt{(1/N)^* \text{ Average Variance} + (1-(1/N)) \text{ Average Covariance}}$

- **For large N, 1/N => 0**
  - — Average variance term associated with unique risks becomes irrelevant !!!
  - — This is fundamentally important: investors do not need worry about uncertainties of individual projects. They can diversify out of them.
  - — Covariance term associated with market risk remains. This is what investors must focus on!

Defining a Formal Measure of Risk

- Investors expect compensation for systematic, undiversifiable (market) risk
- Standard deviation of returns reflects market & unique risks
- Need method to extract market portion of risk
- Define reference point: the market portfolio (MPf), which is the full set of available securities

  r_m = Expected return for MPf

  σ_m = Standard deviation of expected returns on MPf

- Beta: index of investment risk compared to MPf:

  $\beta_i = \rho_{i,m} \sigma_I / \sigma_m$

What Does Beta Imply?

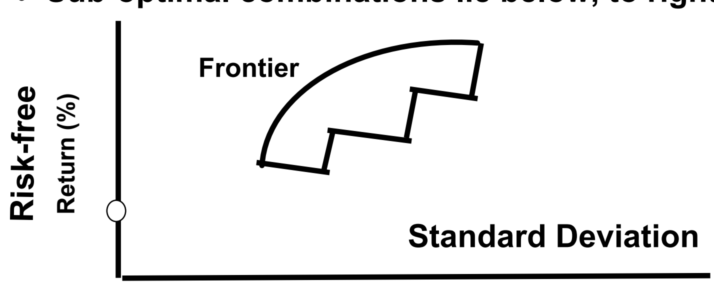
- By definition, the market portfolio has beta = 1.0
- Beta describes the relative variability of returns
  - — Concerned with correlated (systematic) portion of returns
  - — If investment amplifies movements in MPf beta > 1
  - — If attenuates, movements in MPf beta < 1
- Greater Beta reflects market risk of an investment
  => higher returns for investments with higher betas

- Beta calculated for either individual investments or portfolios
- Portfolio beta = weighted average of individual betas

Efficient Frontier for Investments

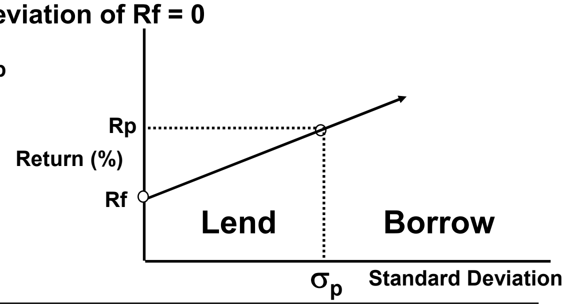
- Example demonstrated role of diversification
- Combinations of many securities result in optimum
  - – Maximum return for given risk level
  - – Minimum risk for given level of return

- Sub-optimal combinations lie below, to right of frontier



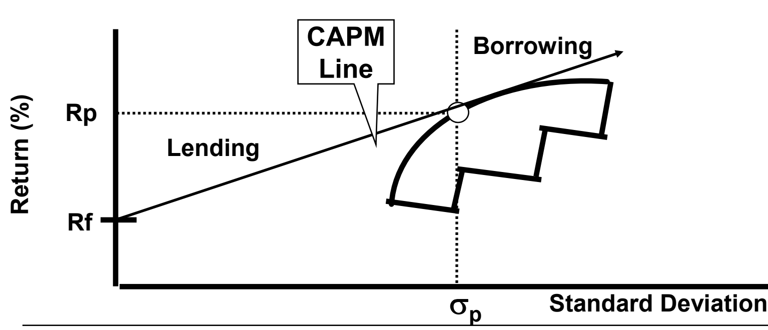
Combining Risk-Free and Risky Investments

- Investors can mix "risky" and "risk-free" investments to balance return and "risk"
- For any combination of risk-free and risky investing
  - — Expected return is weighted average of risk-free (Rf) and portfolio return (Rp)
  - — Standard deviation of Rf = 0
  - —$\sigma_{mix} = x_p \sigma_p$



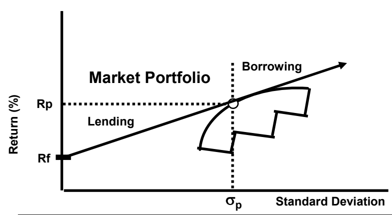
CAPM Defines Premium due to Risk

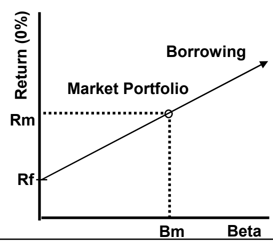
- The line representing best returns for risk is the CAPM line
- This is crux of Capital Asset Pricing Model -- it gives price (risk premium) for assets



Determining Discount rate for Individual Investments

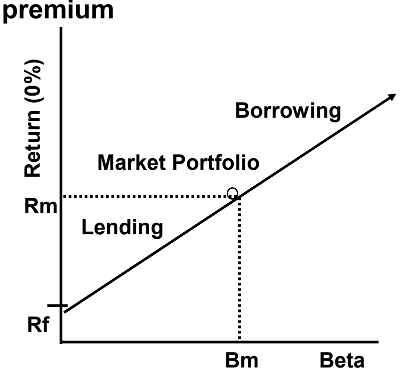
- CAPM models maximized expected return
- Beta indexes risk of individual investment to market portfolio
- Market portfolio is tangent point in CAPM
- Relation between beta and individual expected return results in:





Relation of Expected Return and Beta

- <u>Security Market Line</u> (SML)
  - — Rp = Rf + Bp*(Rm - Rf)
  - — Rm is expected return of market portfolio
  - — Rm - Rf is the market risk premium
  - — Bp = beta of investment to be evaluated
- For the market portfolio, Bm = Bp = 1
- For other investments, expected return scales with Bp



Implementing the CAPM: From Theory to Project Evaluation

- **Theory: Project discount rate should be based on project beta**
  - — Investors can diversify away unique project risks
  - — Adjustment apparent if project is carbon-copy of firm (McDonald's #10,001) ==> WACC applies
- **Practice: adjustment not trivial on most projects**
  - — Consider past experiences, returns in comparable industries
  - — Detail unique aspects of specific project
  - — Apply information to adjust discount rate

A General Rule for Managers

- **CAPM translates to a simple rule:**

Use risk adjusted discount rate to calculate NPV for projects, Accept all positive NPV projects to maximize value

- **Shareholders can avoid unique risks by diversifying, holding multiple assets**
- **If projects valued properly, wealth is maximized**

Difficulties in Practice

- Estimating project beta may not trivial
- Budget constraints conflict with positive NPV rule
- Employees worry about unique project risks
  - — Career can be adversely affected by bad outcomes
  - — Generally cannot diversify (limited to few projects)
  - — Issue might be addressed through proper incentives
- Reliance on past results to dictate future choices
- Individuals, companies are often “risk positive”
  - — Entrepreneurs
  - — Sometimes may “bet the company”

Summary

- CAPM adjusts discount rates for uncertainty
  - — Models maximum expected return for level of “risk”
  - — Based on observations of securities markets
- Unique “risks” can be diversified
- Investors expect compensation for “market risk”
- Standard deviation of returns reflects both market & unique risks
- Beta is index of market part of investment risk
- Security Market Line relates expected return to beta
  - —Rp = Rf + Bp*(Rm - Rf)
- Moving from theory to practice can be problematic