
APPLIED SYSTEMS ANALYSIS

Engineering Planning and Technology Management

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CHAPTER 11

COMPARISON OVER TIME

11.1 THE PROBLEM

Many projects, particularly large systems, evolve over a long time. Costs incurred in one period generate benefits for many years. The evaluation of whether these projects are worthwhile must therefore compare benefits and costs that occur at quite different times.

The essential problem in evaluating projects over time comes from the fact that money has a time value. A dollar now is not the same as a dollar later. The money represents the same nominal quantity, to be sure, but a dollar later does not have the same usefulness or real value as a dollar now.

The problem is one of comparability. Since a dollar now and a dollar later are not the same, we cannot estimate total benefits (or costs) simply by adding up the benefits (or costs) that occur at different times. That would be like adding apples and oranges. To make a valid evaluation, we need to translate all costs and benefits into comparable amounts.

From a formal mathematical point of view the solution to this problem is simple. It consists of a handful of formulas that depend on only two parameters. The formulas can easily be estimated by hand calculator and are routinely embedded in spreadsheet programs available on personal computers. Their results have also traditionally been displayed in extensive tables, which are standard features of engineering handbooks. This chapter presents and explains these essential formulas.

From a practical point of view the solution is delicate, however. The values generated by the formulas are quite sensitive to their two parameters, the

duration or "life" of the project, N , and—most particularly—the discount rate, r . Furthermore, these factors cannot be known precisely. They are therefore somewhat arbitrary. This reality transforms the problem of evaluating projects over time into something of an art. Chapter 12 presents methods for estimating the discount rate, and Chapter 13 presents the various ways the results of the simple formulas can actually be used in evaluations of projects.

11.2 DISCOUNT RATE

A dollar now is worth more than a dollar in the future because it can, in general, be used productively between now and then. At a personal level, for example, you can place money in a savings account and get a larger amount back after a while. In the economy at large, businesses and governments can use money to build things, grow food, educate people, and do other worthwhile activities.

Additionally, any given amount of money now is typically worth more than the same amount in the future because of inflation. As prices go up due to inflation, the money you have now will buy less and less. The \$100 you have now will, so long as there is inflation, buy more food and clothing than \$100 a year or more from now.

The *discount rate* represents the way money now is worth more than money later. It determines by how much any future amount is discounted, that is, reduced, to make it correspond to an equivalent amount today. The discount rate is thus the key factor in the evaluation of projects over time. It is the parameter that permits us to compare costs and benefits incurred at different times.

The discount rate is generally specified as a rate, given as some percent per year. Normally this rate is assumed to be constant for any particular evaluation. Because we usually have no reason to believe it would change in any known way, we take it to be constant over time when looking at any project. It may, however, be quite different for various individuals, companies, or governments, and may also vary from any person or group as circumstances change. Chapter 12 indicates how to choose the appropriate discount rate.

The discount rate is similar to an interest rate, but is actually quite a different concept. It is similar in that both can be stated in percent per period, and both can indicate a connection between money now and money later. The difference is that the discount rate represents real change in value to a person or group, as determined by their possibilities for productive use of the money and the effects of inflation; the interest rate narrowly defines a contractual arrangement between a borrower and a lender. This distinction implies a general rule:

$$\text{discount rate} > \text{interest rate}$$

Indeed, if people were not getting more value from the money they borrow than the interest they pay for it, they would be silly to go to the effort and nuisance of incurring the debt. Chapter 12 on the choice of discount rate goes into this issue in detail. For the moment it is sufficient to recognize that the discount rate is mathematically similar to an interest rate.

11.3 FORMULAS

Four formulas cover the basic range of situations an analyst should know in order to compare money at different times. Each corresponds to one of the four situations defined by whether one is dealing with a unique amount of money or a series of constant periodic revenues or costs. Table 11.1 defines the position of these four formulas.

Only two formulas need to be remembered in practice. This is because the formulas for going from the present to the future are the inverse of those going from the future to the present. This becomes evident below, as each formula is defined.

The notation for the formulas is defined by Figure 11.1 and the following:

- The Present Amount, P , is either money spent or received now, or the value now of future sums.
- The Future Amount, F , is some amount in the future N periods from now.
- The Series of Equal Amounts, R , is a constant stream of equal amounts received at the end of each of N periods.
- The Period is the fixed interval of time for which the discount rate is defined. It is typically, but not necessarily, a year.

Compound amount formula. This defines the future value of a given amount at present after N periods:

$$F = P(1 + r)^N$$

where r is given as a decimal (for example, $r = 10\%$ is stated as $r = 0.10$). This is also the formula used to calculate the way interest compounds, as in a bank account. It is simply a direct extension of the way money changes in value over time: after the first year P is worth r percent more, for a total value of $P(1 + r)$; after the second year this grows by another r percent to $P(1 + r)^2$, and so on.

The *compound amount factor* is simply the quantity that defines the equation: $(1 + r)^N$. Table 11.2 provides a few interesting values of this factor, along with those of other factors defined as follows.

TABLE 11.1
Role of the four formulas for the evaluation over time

Moment in time	Types of amounts	
	Unique	Constant series
Present to future	Compound amount	Capital recovery
Future to present	Present value	Series present value

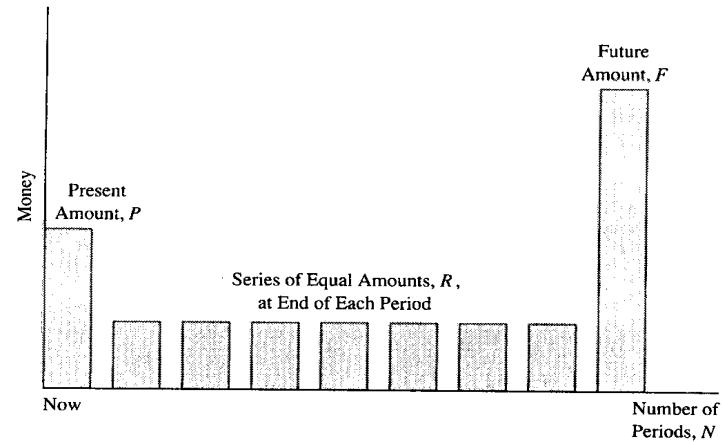


FIGURE 11.1
Illustration of terms used in formulas for comparing money over time.

TABLE 11.2
Summary tables of factors used in evaluations over time for various periods and discount rates

Number of periods	Compound amount at			Capital recovery at		
	5%	10%	15%	5%	10%	15%
5	1.28	1.61	2.01	0.231	0.264	0.298
10	1.63	2.59	4.06	0.130	0.163	0.199
15	2.08	4.18	8.14	0.0963	0.131	0.171
20	2.65	6.72	16.4	0.0802	0.117	0.160
50	11.5	117	1084	0.0548	0.101	0.150

Number of periods	Present value at			Series present value at		
	5%	10%	15%	5%	10%	15%
5	0.783	0.621	0.497	4.33	3.79	3.35
10	0.614	0.386	0.247	7.72	6.14	5.02
15	0.481	0.239	0.123	10.4	7.61	5.85
20	0.377	0.149	0.0611	12.5	8.51	6.26
50	0.087	0.0085	0.0009	18.3	9.91	6.66

Present value formula. This defines the present value of a future sum at N years. It is simply the inverse of the compound amount formula:

$$P = F(1 + r)^{-N}$$

The *present value factor* is the quantity that defines this formula:

$$(1 + r)^{-N} = (\text{compound amount factor})^{-1}$$

(See box for application of the above two formulas.)

Capital recovery formula. This defines the stream of N constant payments R that are equivalent to a present sum P . It might, for example, be an annuity one could buy.

This formula is a bit tricky to derive, but ends up quite simply. It is easiest to determine first the value of the future sum that is equivalent to the series. This is:

$$S = \sum R(1 + r)^i = \frac{R[(1 + r)^N - 1]}{r}$$

where the far right hand expression is the formula for the sum of a geometric series, as given by any mathematical handbook. The equivalent present value is obtained by dividing by $(1 + r)^N$. After rearranging the terms, this results in

$$R = \frac{P[r(1 + r)^N]}{[(1 + r)^N - 1]}$$

The *capital recovery factor*, crf , is the complicated expression of r and N above. Thus:

$$R = P(crf)$$

Compound Amount and Present Value

Suppose you buy \$100 worth of Government Savings Bonds that pay 5% a year for your sister's new baby. What will this child be able to cash them in for in 20 years, when it is time to go to college? Using the compound amount formula we get from Table 11.2

$$\begin{aligned} \text{Future Amount} &= \text{Present Amount} (1 + 0.05)^{20} \\ &= 100(2.65) = \$265 \end{aligned}$$

Conversely, suppose you propose to buy a "zero" bond, a certificate that simply promises to pay its face value at maturity. What would be the proper value now for a \$1000 "zero" bond in 20 years, using a discount rate of 10%?

$$\begin{aligned} \text{Present Amount} &= \text{Future Amount} (1 + 0.1)^{-20} \\ &= 1000(0.149) = \$149 \end{aligned}$$

Note that the capital recovery factor converges to r for long periods. As N is large, $(1 + r)^N \gg 1$, and then

$$crf \sim r \quad \text{for large } N$$

This is evident in Table 11.2; looking across at 50 periods, one sees that the capital recovery factor approximates the discount rate.

Series present value formula. This defines the present value of a constant series of payments R . It is simply the inverse of the preceding. Thus

$$P = R \left(\frac{1}{crf} \right)$$

The *series present value factor*, the multiplier of R to obtain P , is thus the inverse of the capital recovery factor, as the formula states. (See box for application of these last two formulas.)

As the reader can now appreciate, the only two formulas that need to be remembered in practice are the compound amount factor and the capital recovery factor. They summarize the essence of establishing comparability between money at different times.

Capital Recovery and Series Present Value

Suppose you are considering the idea of paying \$5000 to increase the insulation of the house in which you plan to live the next 50 years. If your personal discount rate is 15%, how much should the annual energy savings be to justify the investment?

$$\begin{aligned} \text{Annual Savings Necessary} &= \text{Present Cost} (crf) \\ &= 5000(0.15) = \$750 \end{aligned}$$

from Table 11.2.

Conversely, suppose someone wishes to buy your business on the installment plan, paying you \$100,000 in 10 yearly payments of \$10,000. If your discount rate is still 15%, how does this offer compare with someone else's bid of \$60,000 right now?

$$\begin{aligned} \text{Present Value of Installments} &= \text{Annual Amount} \left(\frac{1}{crf} \right) \\ &= 10,000(5.02) = \$50,200 \end{aligned}$$

from Table 11.2.

The \$60,000 offer is far superior, even though the installment plan offers a larger nominal amount.

11.4 APPROXIMATIONS

Two approximations of the formulas are useful. They permit the analyst to determine the relative importance of present and future amounts, without reference to tables or computers. They help determine which elements of a project are critical, and which can be neglected.

Exponential approximation. This formula exploits the fact that the compound amount factor tends toward the exponential as the size of the periods becomes smaller, the compounding period becomes smaller, and the compounding becomes continuous rather than periodic:

$$(1 + r)^N \sim e^{rN}$$

Use of Rule of 72

This approximation is useful when reasonable estimates have to be made on the spot. Such was the case when the author was participating in a discussion of the Board of Directors for the Massachusetts Port Authority. The topic, in 1986, was the future size of the major Boston airport. The current traffic was about 24 million passengers a year, and some consultants had just argued that it was reasonable to expect that the traffic in the year 2010 would be 40 million.

The Chairman of the Board turned to me as an independent expert and asked first if I considered the forecast reasonable. My thinking was as follows:

The forecast is for 24 years hence. If the current traffic doubled in that time, to 48 million passengers, the compound rate of growth would be $\frac{72}{24} = 3\%$. But as the forecast was for significantly fewer than 48 million passengers, the implied rate of growth is less than 3%, probably about $2\frac{1}{2}\%$ (it is actually about $2\frac{1}{4}\%$). My answer on the spot was this: that "the implied rate of growth was far less than half the historical average, and thus probably unreasonable."

The Chairman then asked when I thought the 40 million forecast could be reached. Reversing gears, I reasoned:

The historical rates of growth in traffic have been about 7% a year, and sometimes as high as 10%. Traffic could then double in $\frac{72}{7} \sim 10$ years or possibly as soon as $\frac{72}{10} \sim 7$ years. Since we are concerned with a traffic level less than double, a couple years off either estimate would be appropriate. So I answered "In about 8 years or so," and, rounding off to a nice figure such as planners like to use, continued "that is, quite possibly around 1995."

The exponential approximation is always somewhat of an overestimate. The continuous compounding implied by the exponential both continually builds up the present value and builds on the increments. The overestimate is typically quite small, however. Over 20 years at $r = 5\%$, for example, the overestimate using the continuous approximation is only about $2\frac{1}{2}\%$. The exponential approximation is thus quite satisfactory for most preliminary evaluations of projects.

Rule of 72 (or 70). The rule of 72 (or 70) derives directly from the exponential approximation. It is based on the fact that

$$e^{0.693} = 2.0$$

This means that, using continuous compounding, a present sum doubles for any combination of $rN = 0.693$. To a first approximation, this doubling rule is taken as $rN = 70$, where r is now thought of as a percent rather than a decimal.

This characteristic is generally ascribed to $rN = 72$. The logical excuse is that the exponential approximation is an overestimate and that, to compensate for this fact, it is desirable to increase the exponent necessary for doubling. The real reason appears to be that 72 can be more easily divided by more numbers than 70, and is thus more convenient (see box).

11.5 SENSITIVITY TO DISCOUNT RATE

The formulas for comparing the value of money at different times are quite sensitive to their two parameters, the discount rate, r , and the life of the project, N . They are especially sensitive to the discount rate.

The value of either of these two parameters is never defined precisely, unfortunately. They are both a matter of judgment. Even if the analyst is required by law or some outside constraint to use particular values of these parameters, they have still been somewhat arbitrarily defined by someone. Professional responsibility therefore implies that the analyst be aware of the sensitivity of the evaluation to both of these parameters.

The present value of future sums decreases rapidly for any reasonable discount rate. (This can safely be assumed to be within 5 to 15% in constant dollar terms, as Chapter 12 discusses in detail.) As Figure 11.2 shows, the dropoff in value is quite steep. The value of \$100 ten years from now is easily worth half that, \$50 or less, depending on the discount rate selected.

The phenomenon has a significant implication for any evaluation. It means that the analyst can almost totally discount both long-term future benefits and costs in any evaluation. As Table 11.2 and Figure 11.2 indicate, the present worth factor becomes less than 1% as soon as about 30 years or so. Unless long-term future benefits or costs are absolutely stupendous in comparison to those of the present, they can really be neglected in the evaluation.

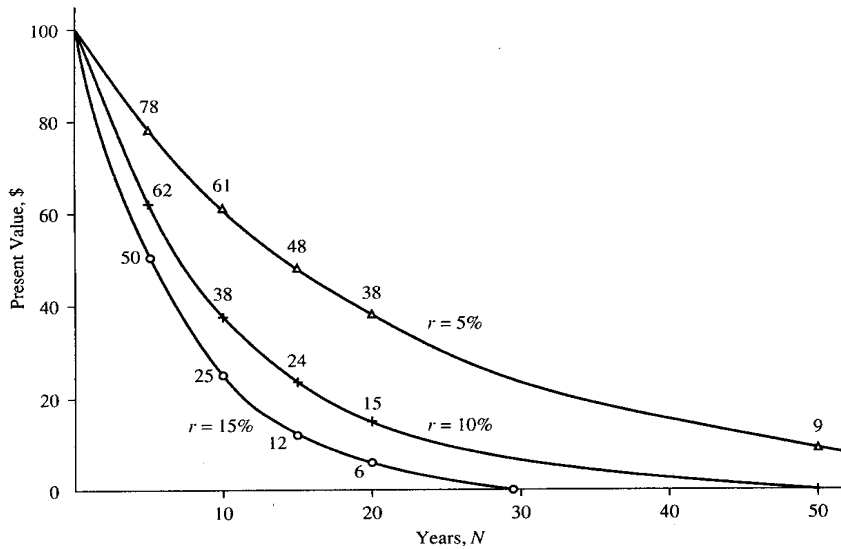


FIGURE 11.2
Present value of \$100, N years from now at discount rates of 5, 10, and 15% per year.

The present value of future sums is also very sensitive to the discount rate that is selected. Choosing a discount rate of 10% instead of 5%, or of 15% instead of 10%, virtually cuts the present value in half.

This fact means that the overall desirability of any project can be very sensitive to the discount rate selected. As can be imagined, and as is discussed in Chapter 12, the consequence is that the selection of discount rates frequently becomes highly politicized. Advocates of particular kinds of projects will fight to define the discount rate to favor their enterprise.

11.6 SENSITIVITY TO PROJECT LIFE

The value of a project is also sensitive to the number of years, N , it provides benefits. Normally, the useful life of an investment can only be estimated if it has a relatively short life, which has been observed frequently. Such is generally the case with machine tools and other mechanical parts. The useful life of large, unique projects, such as power plants or refineries, is quite uncertain. Often the useful life of a project is defined arbitrarily.

In practice, the overall value of a project is not especially sensitive to the length of life selected for it. This is because the evaluation is already particularly sensitive to the discount rate. Normally, the longer the useful life of a project,

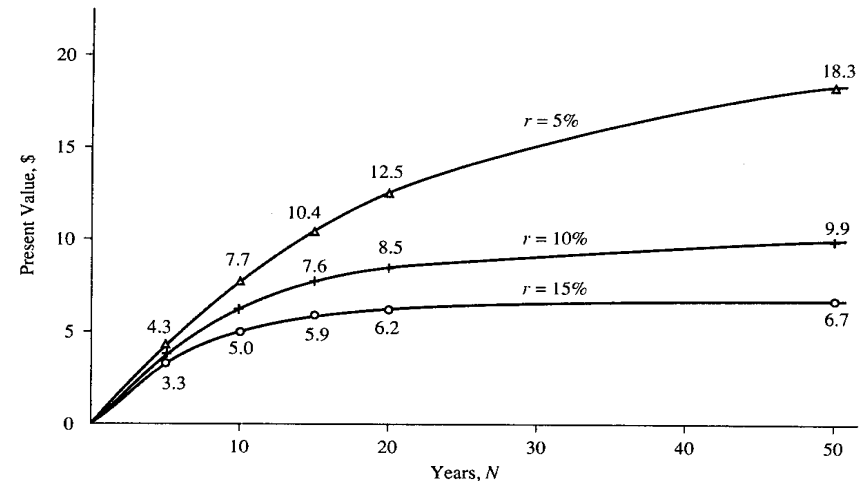


FIGURE 11.3
Present value of a series of \$1 per year for N years, at discount rates of 5, 10, and 15% per year.

the more benefits are included in an evaluation and the better the project. The overall increase is generally relatively small, however.

The relative insensitivity to project life is illustrated by Figure 11.3, which plots the cumulative present value of a constant stream of money for N years, as given in Table 11.2. As can be seen, the rate of increase is essentially flat beyond 20 years, for discount rates of 10% and more. The net increase in value obtained by increasing the project life from 20 to 50 years is only about 15%. This is not enough to alter the desirability of a project enormously.

The length of a project's life is only really important when the discount rate is low. Thus when $r = 5\%$, the increase in present value by increasing the project life from 20 to 50 years is about 50%. This is significant. This sensitivity is due, as can be seen by looking at Figure 11.2, to the fact that the value of future money does not drop off to zero for 5%. This is another reason that people fight for a low discount rate—it makes long-lasting projects, such as railroad lines or dams, look more attractive.

11.7 APPLICATION

To illustrate the formulas presented in Section 11.3, and the sensitivity to discount rate and project life, consider the example portrayed in Figure 11.4. This graph shows the *cash flow* of revenues and costs for a project, that is, the sequence through time of the flow of money into and out of an organization.

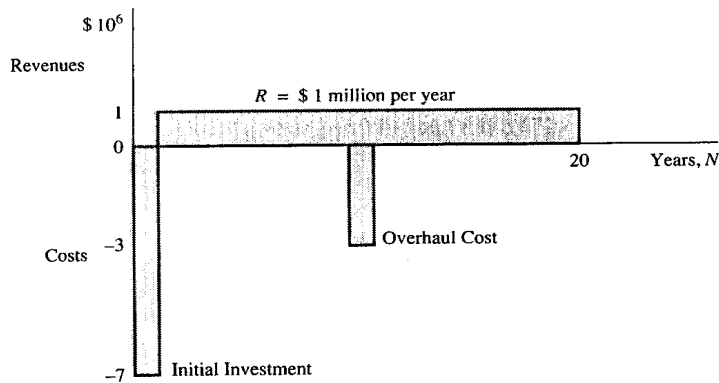


FIGURE 11.4
Cash flow of revenues and costs for example project.

This project is fairly typical. It involves a substantial initial investment to produce a stream of benefits for some time. It also involves some substantial future cost, as for a complete overhaul of the system. This might be, for instance, the acquisition of a ship.

The revenues and costs of the project are as shown in Figure 11.4: revenues of \$1 million a year for 20 years, totalling \$20 million; costs of \$7 million initially and \$3 million in 10 years, totalling \$10 million.

If we were, wrongly, just to look at the nominal or stated revenues and costs, the project looks good. The nominal revenues are twice the nominal costs and this could be interpreted as a 100% profit. The difficulty is that since money has a time value, comparing money at different times on an equal footing is invalid. The future costs and benefits should be properly discounted.

For the example, we use a discount rate of 10%, which is a reasonable value to use if all future revenues and costs are stated in constant dollars (see Chapter 12 for a detailed discussion).

To place all future sums on the same footing, regardless of when received, we discount them to the present. In doing so we are calculating the present value of all future cash flows, which business people refer to as the *discounted cash flow* of the project.

The present value of the costs is the sum of the initial investment, undiscounted since it occurs now, and of the future cost of overhaul, discounted using the present value factor for 10% and 10 years, given in Table 11.2:

$$\text{Present Value Costs} = 7 + 3(0.386) = \$8.158 \text{ million}$$

The present value of the series of uniform revenues is given by the series value factor for 20 years at 10%:

$$\text{Present Value Revenues} = 1(8.51) = \$8.51 \text{ million}$$

Given these two results we see that the revenues are slightly larger than the costs, when all costs and revenues are placed on a comparable basis. The difference between these two values, the benefits minus the costs, is the *net present value* or NPV:

$$\text{Net Present Value} = 8.51 - 8.158 = \$0.352 \text{ million}$$

Since the NPV is positive, the project is marginally worthwhile, at least when a discount rate of 10% is used.

This result is quite sensitive to the discount rate, as can be seen by redoing the sums with a discount rate of 15%. We then have

$$\text{Present Value Costs} = 7 + 3(0.247) = \$7.741 \text{ million}$$

and

$$\text{Present Value Revenues} = 1(6.26) = \$6.26 \text{ million}$$

Therefore

$$\text{Net Present Value} = 6.26 - 7.741 = -\$1.481 \text{ million}$$

The project now does not appear worthwhile since NPV is negative. On the basis of present values, costs now exceed revenues. The higher discount rate, which makes all future money less significant, particularly affects the revenues.

Finally, the result is not especially sensitive to the life of the project. Suppose that the revenues of the project proceed for 50 years, with no further overhaul or other cost. We then obtain, using a discount rate of 15% again,

$$\text{Present Value Revenues} = 1(6.66) = \$6.66 \text{ million}$$

Although we have increased the life of the project by 150%, and added \$30 million in nominal revenues, the present value of the revenues has only increased by \$0.4 million, or a mere 6%.

11.8 OPTIMAL PROJECT SIZE

The fact that money has a time value has important implications for the optimal size of a project. This section explains the phenomenon and provides guidelines for determining the optimal size of a project.

The direct implication of the time value of money is that investments should be deferred until needed. There is no sense in sinking capital into a project—and thus paying interest on a loan or losing out on the profits this money would otherwise provide—unless there are substantial benefits that compensate for the time cost of the money. Thus, in general, designers should look for ways to build up a system incrementally, providing extra facilities just in time, when actually needed.

The incentive to defer projects can be counterbalanced by many factors, of course. Some facilities can only be provided in specific chunks, and must be

built large if at all. Runways for an airport generally fall into this category, for example. At other times the systems designers may face a unique opportunity to proceed and cannot afford to waste it. Having received permission to build a power plant say, planners have a strong incentive to proceed, to avoid the possibility that future politics might change the rules.

Economies of scale (see Sections 2.4 and 4.5) provide the major incentive to build larger projects and to build them in anticipation of need. The existence of economies of scale means that it is cheaper, per unit of addition, to build large rather than small. This incentive is directly opposite to that of the time value of money, which is to build small increments only as needed.

The effects of economies of scale and the time value of money balance each other to define an optimal size of a project for any situation. This calculation naturally depends on the specifics of a particular case, but it is possible to derive general guidelines. The procedure is to define a general expression for the total present value cost of a system, and to minimize it as a function of the size of the project.

To illustrate the way to define the optimal size of a project, consider a basic case. Assume that a system needs to have a constant increase in capacity each year, G . The designers can provide this in small amounts, or in bigger additions that would satisfy N years of the requirement. From Section 4.5, the general cost function for GN units of capacity is: $C(GN) = A_0(GN)^a$. The present value of the total costs of all the additions built to infinity in N year chunks, PV_N , can usefully be stated as the cost of the first increment plus the present value of all future additions starting with the next one built N years hence:

$$PV_N = A_0(GN)^a + e^{-rN}PV_N$$

The advantage of this way of stating the present value is that it reduces simply to

$$PV_N = \frac{A_0(GN)^a}{(1 - e^{-rN})}$$

The optimal size of the project is defined by the value of N that minimizes the present value of all costs.

The optimal project size, defined by the optimal interval, N^* , can be derived as above. For our particular case, it is given implicitly by the formula:

$$\frac{rN^*}{(e^{rN^*} - 1)} = a$$

This can be understood by looking at Figure 11.5. This shows the way the effects of economies of scale and the discount rate balance each other. For higher discount rates, the optimal interval and project size decreases. Conversely, when economies of scale are more important (lower a), the optimal project size increases. Note that for typical values of the discount rate around 10% (see

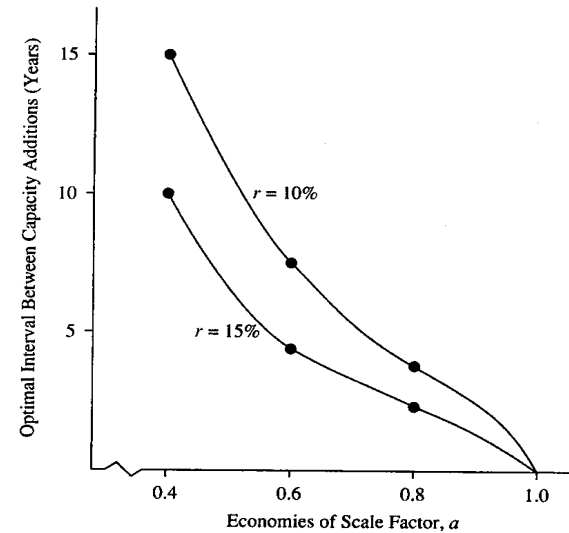


FIGURE 11.5

The optimal project size as defined by a balance between the discount rate, r , and the economies of scale.

Sections 12.4 and 12.5), the economically optimal project size should cover only about 5 to 10 years of growth even for large economies of scale.

REFERENCE

ATT Company, (1977). "Economic Study Techniques," Chapter 12, *Engineering Economy*, (3rd ed.), McGraw-Hill, New York.

PROBLEMS

11.1. Asphalt and Concrete Pavements

A local government faces the classic dilemma of choosing between a design with high first cost and longer life and one with a lower first cost but shorter life: concrete versus asphalt pavements for an access road. The projected first cost per square yard of roadway is \$21.00 for the concrete pavement and \$17.80 for the asphalt.

Either pavement would have to be resurfaced eventually with asphalt, but at different times. The best information is that the concrete base was to be resurfaced every 27 years, and the asphalt base every 17 years. Annual maintenance costs for either roadway will be \$8000.

- (a) Assuming a 10% real discount rate, how much cheaper would the more expensive material have to become to be competitive on this job?
- (b) At what rate or interest are the two costs equal?
- (c) Which solution would you recommend if (1) the national government pays 50% of the initial costs but none of the maintenance or replacement costs (a typical kind of subsidy) or (2) the local government sells municipal bonds that, because of tax advantages, only have to pay 6% real interest?

11.2. *New Car*

A salesperson is shopping for a new automobile and is comparing the benefits and costs of various models that he or she is considering. One such model has the following costs and benefits that are approximated to occur at the end of the year. The salesperson's policy is to get a new car every three years.

Year	Cost (\$)	Benefits (\$)
0	10,600	
1	1600	6000
2	2000	5600
3	2400	7000

Using $r = 5%$, what is the present value of benefits? Of costs?

11.3. *Patent Sale*

An inventor contracted to give a corporation the exclusive right to use a patent. The corporation agreed to pay the owner \$1000 a year at the end of each of the first 4 years while developing a market for the invention; \$5000 at the end of each year for the next 8 years; and \$2000 at the end of each year for the final 5 years of the 17-year life of the patent. After two years, the corporation wishes to buy the patent outright: what is the maximum the corporation could afford to pay the owner at this time if it requires a 16% rate of return on investments before income taxes?

11.4. *Trust Fund*

Reed Ekasch was born on July 1, 1967. On July 1, 1968, an aunt started a trust. She then paid \$1000 each July 1 up to and including 1979—a total outlay of \$12,000 by the aunt. One purpose of the trust was to help finance Reed's college education: it was provided that \$2500 should be withdrawn for this purpose each year for four years, starting with the boy's 18th birthday, July 1, 1985—a total withdrawal of \$10,000. The remainder of the trust is to accumulate until Reed is 30 years old on July 1, 1997, to help him finance the purchase of a home. If the trust earns 6% compounded annually after taxes, how much will Reed receive on his 30th birthday?

11.5. *Ace Woodworking, Ltd.*

AWL is thinking of computerizing its order processing system. It currently spends \$2.5 million/year doing it by hand. For the 10-year life cycle of the system, the two bids are analyzed as follows:

System	Costs (in current \$, 10 ⁶)			
	Initial	Repair	Maintenance	Operating
IBM	16	included	included	included
HAL	4.5	(five years from now)	1/year	0.5/year

Which system should the company choose if its real annual discount rate is: 5%? 10%? 15%?

11.6. *XYZ Corporation*

The production department of XYZ Corporation is thinking of buying one of two numerically controlled zipper machines. Either would last 10 years. Additionally, the costs, in $\$ \times 10^3$, are:

	Machine X	Machine Y
Initial Investment	20	30
Salvage Value	5	7
Annual Operating Cost	12	6
Annual Maintenance Cost	2	4

Assume that the annual costs are paid at the end of each year and the salvage value at the end of the tenth year. Using a discount rate of 15%:

- (a) What is the present worth of the cash flow for machine X? Machine Y?
- (b) Which machine should be purchased?

11.7. *Ren O'Vait*

The famous architect, Ren O'Vait, has bought a dilapidated Victorian mansion. In addition to various structural and cosmetic defects, the heating system needs replacement. Ms. O'Vait must choose between a conventional gas-fired furnace and a solar heating system with a small backup gas heater. Based on her preliminary designs, she estimates the following costs:

	Gas	Solar/gas
Initial cost	\$5000	\$15,000
Annual nonfuel	100	300
Annual fuel costs	1400	200

- (a) Assuming a 20-year design life for both systems, and a discount rate of 8%, which system is cheaper?
- (b) Which system would be preferred if the discount rate were 5%? 10%?
- (c) Which system will be favored if, with a discount rate of 8%, the time horizon is reduced to 15 years? Extended to 25 years?

- (d) How much would the price of gas have to change to make the two options equally attractive?
- (e) If the costs of home improvements are deductible from taxable income, which system will be favored?

11.8. Snowbird High

On December 1, the superintendent of schools finds out that the heating plant in Snowbird High will not last through the season. The boiler and firebox must either be repaired extensively or replaced during the December holidays. There are three choices:

- The present coal-fired system can be completely overhauled at a cost of \$40,000. It should then last another 20 years, 5 years beyond the time when the building is to be torn down and replaced. Annual heating cost with this system will be about \$5000.
- A new gas-fired boiler can be installed. It is a prefabricated unit with an estimated useful life of 10 years. It costs only \$20,000 to install but \$6000 per year to operate.
- A residual oil unit could heat for only \$3500 per year. The unit would last at least 30 years and would cost \$65,000.

None of the above units could be salvaged if the building is abandoned.

What is the best solution for a discount rate of 7%? 10%?

11.9. Hi-Tacky PC's

You have been working hard in graduate school and decide you should purchase your own personal computer. Two types are available:

- Brand PC costs \$2500, will last for 9 years, and comes with a yearly maintenance fee of \$100.
- Hi-Tacky PC costs \$1600, has no maintenance fee, but its hard disk must be replaced every 3 years at a cost of \$1000.

Over 9 years and using a 10% discount rate in your calculations, which PC should you purchase based on Net Present Value?

11.10. Balloon Payment

Normally, the Ready-Tech company sells its blood analysis machines to labs on long term "Regular" leases: \$10,000/year for 10 years. The sales manager suggests an "Easy Start" program of \$6000/year for the first 5 years followed by "balloon payments" of \$18,000/year for the next 5. "We'll take in \$20,000 more!" he says, "We may even double our net profit!"

- (a) If Ready-Tech's discount rate is 20%, is "Easy Start" better than "Regular" for the company?
- (b) How would the situation change if the initial payments under "Easy Start" were \$7000/year?

11.11. Size That Project!

- (a) The airport authority needs to process 1000 more passengers an hour at the peak periods, each year. It estimates that this requires an extra 1000m²

- annually. What size project is most economical if its discount rate is 10% and it realizes moderate economies of scale in construction, $C(Y) = A_0Y^{0.7}$?
- (b) The municipal water board needs to add 50 acre-feet of reservoir capacity each year, on average. It has strong economies of scale in constructing the tanks, $C(Y) = A_0Y^{0.5}$; and has a low discount rate since it raises money through low yield municipal bonds, $r = 6\%$. How big should it build the next reservoir?
- (c) Same as (b), but for a private company that must get money from its banks, $r = 12\%$.
- (d) A trucking company must acquire 50,000 m² of warehouse space a year. Its discount rate is 15% and it hardly sees any economies of scale, $C(Y) = A_0Y^{0.9}$. How many years in advance should it build?