Airside Delays and Congestion

Amedeo R. Odoni
Massachusetts Institute of Technology

Airside Congestion

- Objectives
  - Introduce fundamental concepts regarding airside delay

- Topics
  - The airport as a queuing system
  - Dynamic behavior
  - Long-term characteristics of airside delay
  - Non-linearity
  - Air traffic flow management
  - Annual capacity of an airport
  - Measuring delay

Reference: Chapters 11, 23
Queues

- Queuing Theory is the branch of operations research concerned with waiting lines (delays/congestion)
- A queuing system consists of a user source, a queue and a service facility with one or more identical parallel servers
- A queuing network is a set of interconnected queuing systems
- Fundamental parameters of a queuing system:
  - Demand rate
  - Capacity (service rate)
  - Demand inter-arrival times
  - Service times
  - Utilization ratio
  - Queue discipline (FIFO, SIRO, LIFO, priorities, etc)

Queuing network consisting of five queuing systems
Delays will occur when, over a time interval, the demand rate exceeds the service rate (“demand exceeds capacity”)

Delays may also occur when the demand rate is less than the service rate -- this is due to probabilistic fluctuations in inter-arrival and/or service times (i.e., to short-term surges in demand or to slowdowns in service)

These “probabilistic” (or “stochastic”) delays may be large if the demand rate is close to (although lower than) capacity over a long period of time.

1. The dynamic behavior of a queue can be complex and difficult to predict.
2. Expected delay changes non-linearly with changes in the demand rate or the capacity.
3. The closer the demand rate is to capacity, the more sensitive expected delay becomes to changes in the demand rate or the capacity.
4. The time when peaks in expected delay occur may lag behind the time when demand peaks.
5. The expected delay at any given time depends on the “history” of the queue prior to that time.
6. The variance (variability) of delay also increases when the demand rate is close to capacity.
Example of the Dynamic Behavior of a Queue

![Graph showing expected delays for different levels of capacity]

Expected delay for four different levels of capacity
(R1 = capacity is 80 movements per hour; R2 = 90; R3 = 100; R4 = 110)

Scheduled aircraft movements at LGA before and after slot lottery

![Graph showing scheduled movements per hour]

Scheduled movements per hour

Time of day (e.g., 5 = 0500 – 0559)
### Behavior of Queuing Systems in the “Long Run”

- The “utilization ratio”, \( \rho \), measures the intensity of use of a queuing system:
  \[
  \rho = \frac{\text{demand rate}}{\text{service rate}} = \frac{\lambda}{\mu} 
  \]

- A queuing system cannot be operated in the long run with a utilization ratio which exceeds 1; the longer such a system is operated, the longer the queue length and waiting time will be.

- A queuing system will be able to reach a long-term equilibrium (“steady state”) in its operation, only if \( \rho < 1 \), in the long run.

### Estimated average delay at LGA before and after slot lottery in 2001

- Average delay (mins per movt)
- Time of day

![Graph showing estimated average delay at LGA before and after slot lottery in 2001](image_url)

- For queuing systems that reach steady state the expected queue length and expected delay are proportional to:
  \[
  \frac{1}{1 - \rho}
  \]

- Thus, as the demand rate approaches the service rate (or as \(\rho \to 1\), or as “demand approaches capacity”) the average queue length and average delay increase rapidly.

- The “proportionality constant” increases with the variability of demand inter-arrival times and of service times.

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Delay vs. Demand and Capacity

![Expected delay vs. demand and capacity graph](image-url)

- Expected delay
- Demand
- Capacity \((\rho = 1.0)\)
High Sensitivity of Delay at High Levels of Utilization

Expected delay versus Capacity

\( \rho = 1.0 \)

Relationship between traffic and en route delay

Source: Eurocontrol PRC (2001)
Some statistics for the dynamic queuing example

<table>
<thead>
<tr>
<th>Capacity (movements/hr)</th>
<th>Maximum of expected waiting time (minutes)</th>
<th>Expected waiting time, all movements (minutes)</th>
<th>Utilization ratio (24 hours)</th>
<th>Utilization ratio (6:00–21:59)</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>2</td>
<td>0.8</td>
<td>0.455</td>
<td>0.664</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>1.6</td>
<td>0.5</td>
<td>0.731</td>
</tr>
<tr>
<td>90</td>
<td>13</td>
<td>4.3</td>
<td>0.556</td>
<td>0.812</td>
</tr>
<tr>
<td>80</td>
<td>39</td>
<td>12.8</td>
<td>0.625</td>
<td>0.913</td>
</tr>
</tbody>
</table>

Total demand = 1200 movements per day

A More “Formal” Setting

In a queuing system with one server, let:

- **X** = a random variable that represents the time between successive arrivals of demands at the system ("inter-arrival times" or "headways between arrivals")
  - \( \lambda \) = rate of demand arrivals per unit of time
  - (and therefore \( \frac{1}{\lambda} = E(X) \) = expected time between demand arrivals = "average headway between arrivals")
  - \( \sigma_X^2 \) = variance of the time between demand arrivals

- **T** = a random variable that represents the time required to service a demand at the queuing system ("service times")
  - \( \mu \) = service rate per unit of time ("capacity")
  - (and therefore \( \frac{1}{\mu} = E(T) \) = expected service time = "average service time")
  - \( \sigma_T^2 \) = variance of service times

- \( \rho = \frac{\lambda}{\mu} \) = “utilization ratio” (an indication of intensity of utilization)
Four Fundamental Measures of Performance

- The quantities of interest:
  - $L = \text{expected number of users in queueing system (includes those in queue and those receiving service)}$
  - $L_q = \text{expected number of users in queue}$
  - $W = \text{expected time in queueing system per user (waiting time plus service time)}$
  - $W_q = \text{expected waiting time in queue per user}$

- 4 unknowns $\Rightarrow$ We need 4 equations

Relationships among the Four Measures in Steady-State

$W = W_q + E(t) = W_q + 1/\mu \quad (1)$

[Note: (1) makes intuitive sense]

$L_q = \lambda W_q \quad (2)$

$L = \lambda W \quad (3)$

[Note: (2) and (3) are far less obvious and are known as “Little’s formulae”.]
An Important Result: The “P-K formula”

For queuing systems with Poisson* demands, ANY type of service time, one server and infinite queuing capacity (M/G/1 system):

\[ W_q = \frac{\lambda \left[ \left( \frac{1}{\mu} \right)^2 + \sigma^2_T \right]}{2(1 - \rho)} = \frac{\lambda \left[ E^2(T) + \sigma^2_T \right]}{2(1 - \rho)} \]

Assumes steady-state conditions: \( \rho < 1 \) (\( \lambda < \mu \))

* A Poisson process consists of the occurrence of a sequence of “events” such that: (a) successive inter-arrival times between events are mutually independent; and (b) event inter-arrival times are all described by the same negative exponential probability density function.

Dependence on Variability (Variance) of Demand Inter-Arrival Times and of Service Times

Expected delay

Demand

\( \rho = 1.0 \)
Variability of Queues

- The variability of delay also builds up rapidly as demand approaches capacity.
- In “steady state,” the standard deviation -- a measure of variability -- of delay and of queue length is also proportional to:
  \[ \frac{1}{1 - \rho} \]
- A large standard deviation implies unpredictability of delays from day to day and low reliability of schedules

Runway Example

- Single runway, mixed operations
- \( E(t) = 75 \) seconds; \( \sigma_t = 25 \) seconds
  \( \mu = \frac{3600}{75} = 48 \) per hour
- Assume demand is relatively constant for a period of time sufficiently long to have approximately steady-state conditions
- Assume Poisson process is reasonable approximation of the arrival process, i.e., can be used to “simulate” the instants when demands occur
### Estimated expected queue length and waiting time

<table>
<thead>
<tr>
<th>( \lambda ) (per hour)</th>
<th>( \rho )</th>
<th>( L_q ) (%)</th>
<th>( L_q ) (% change)</th>
<th>( W_q ) (seconds)</th>
<th>( W_q ) (% change)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.625</td>
<td>0.58</td>
<td></td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>30.3</td>
<td>0.63125</td>
<td>0.60</td>
<td>3.4%</td>
<td>71</td>
<td>2.9%</td>
</tr>
<tr>
<td>36</td>
<td>0.75</td>
<td>1.25</td>
<td></td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>36.36</td>
<td>0.7575</td>
<td>1.31</td>
<td>4.8%</td>
<td>130</td>
<td>4%</td>
</tr>
<tr>
<td>42</td>
<td>0.875</td>
<td>3.40</td>
<td></td>
<td>292</td>
<td></td>
</tr>
<tr>
<td>42.42</td>
<td>0.88375</td>
<td>3.73</td>
<td>9.7%</td>
<td>317</td>
<td>8.6%</td>
</tr>
<tr>
<td>45</td>
<td>0.9375</td>
<td>7.81</td>
<td></td>
<td>625</td>
<td></td>
</tr>
<tr>
<td>45.45</td>
<td>0.946875</td>
<td>9.38</td>
<td>20.1%</td>
<td>743</td>
<td>18.9%</td>
</tr>
</tbody>
</table>

** Can also estimate the practical hourly capacity (≈ 40.9 per hour) or other LOS-related capacities

### Tools for Estimating Delays

- The estimation of delays at an airport is usually sufficiently complex to require use of computer-based models
  - Dynamic queuing models: solve numerically the equations describing system behavior over time
  - Simulation models (e.g., TAAM, SIMMOD)
- For very rough approximations, simplified models may sometimes be useful
  - Simple (“steady-state”) queuing models
  - Cumulative diagrams
Alternatives for Relieving Airport and Terminal Airspace Congestion

1. Increased airport capacity
   - second, third, … airports
   - new, larger airports to replace older ones
   - more runways, etc. at existing ones
   - improved ATM

2. Demand management
   - total operations
   - by time-of-day

3. Air traffic flow management (ATFM): reduces cost and impact of unavoidable delays

4. Substitute other modes of transportation; use substitutes for transportation (communications); forego travel altogether

Definition and Fundamentals of ATFM

- ATFM: Strategic planning and implementation of regional and national flows to best “match” demand with available capacity and minimize impact of congestion on users and operators of ATM system
- Now vital element of both European and US ATM systems
- Basic steps:
  1. Prediction of potential overloads
  2. Development of strategies
  3. Implementation of strategies
- The tools for accomplishing (2) and (3):
  - Ground-holding (more “strategic”)
  - Re-routing, metering, speed control, sequencing
- Time horizon ranges from months to ~30 minutes
Ground Delay Programs require more advance planning than other ATFM tools

Hanowsky (2006)

Reduced Capacity at SFO Typically Leading to Initiation of a GDP

VFR Conditions
52 A/C Per Hour

Low Ceiling Conditions
32 A/C Per Hour

4-5 nmi spacing
Collaborative Decision-Making (CDM) in U.S.

- New approach to traffic flow management
- Airline Operations Centers (AOCs) and FAA share information on capacities, delays, cancellations, preferences, etc.
- First experiments with GDPs at SFO and EWR (1/98); adopted for all airports (9/98)
- Has saved $$$ in delay costs
- Opportunity to work and make decisions in real time with a common data base
- Greatly expanded scope and objectives at this time

Dynamic Slot Assignment System under CDM

1. FAA estimates airport acceptance rate (AAR) at arrival airport
2. FAA assigns slots to airlines according to AAR on first-scheduled, first-served basis (“ration by schedule”)
3. Each airline tells FAA how its own slots will be used (substitutions and cancellations)
4. After cancellations become known, “compression” is performed to take advantage of empty slots and some flights are moved in their order as a result of “slot credit substitution”
5. FAA assigns controlled time of arrival (CTA) to each flight and an associated controlled time of departure (CTD)
6. (Future?) No CTDs: airline determines take-off time for each flight to meet that flight’s CTA.
CDM Rules for Flight Substitutions and Cancellations

- **Substitutions:**
  - Airlines may freely substitute within their own flight schedule and can move any flight to a slot which is not earlier than that flight’s ETA
- **Cancellations** (the “Slot Credit Substitution” rule):
  - An airline that cancels a flight has the right to advance later flights to the first feasible slot which becomes available as a result of the cancellation.

Schematic of GDP under CDM

- **Ration by Schedule (RBS)**
- **Cancellations and substitutions**
- **Compression**

- ↑ (FAA)
- ↑ (Airlines: AOC)
- ↑ (FAA)
Example of a GDP under CDM

- 30 arrivals per hour; AAL 290 is cancelled

Examples of current, CDM-motivated problems

- FAA: What airport acceptance rates (AAR) to specify over a period of time for any given airport? *How to deal with uncertainty re AARs?*

- Airlines: How many and which flights to delay or cancel?

- FAA: How to ensure the most equitable distribution of benefits among users?

- Airlines and FAA: Collaborative routing.


**Predictability and Effective Control Curves**

*Source: Dr. J. Evans, MIT Lincoln Labs*

- **Control**
- **Prediction**
- **Balance Point of Knowledge and Action**
- **Time**

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**Annual Airside Capacity**

- The number of aircraft movements that can be handled at a reasonable level of service in one year
- Vaguely defined, but very important for planning purposes
- Runway system is typically the limiting element
- Estimation of annual capacity must consider:
  - Typical hourly (saturation) capacity
  - Pattern of airport use during a day
  - Reasonable level of delays during busy hours of day
  - Seasonal and day-of-the-week peaking patterns of demand
**Annual Airside Capacity: Boston Example**

1. Typical hourly runway capacity (based on CCC) = 115.
   Compute: \( A = 115 \times 24 \times 365 = 1,007,400 \)

2. Equivalent of ~16–17 hours of strong activity per day.
   Compute: \( 1,007,400 \times \left(\frac{16}{24}\right) = 671,600 \)

3. ~85% utilization in busy hours for (barely) tolerable delays
   Compute: \( 671,600 \times 0.85 = 570,860 \)

4. Summer season days have about 15% more movements than winter season days
   \( \frac{570,860}{2} + \frac{570,860}{2} \times \frac{1}{1.15} \approx 534,000 \)

This is a *rough estimate* of the ultimate capacity of Logan airport, without expansion of capacity.

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**Annual Capacity Coverage Chart: Boston/Logan**

![Annual Capacity Coverage Chart](image)
### Peaking Characteristics of 80 Airports in ACI Survey (1998)

<table>
<thead>
<tr>
<th>Total annual pax (million)</th>
<th>Sample size</th>
<th>Average monthly peaking ratio*</th>
<th>Range of monthly peaking ratios</th>
<th>Monthly peaking ratios greater than 1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;20</td>
<td>23</td>
<td>1.18</td>
<td>1.09 – 1.43</td>
<td>6 of 23 (26%)</td>
</tr>
<tr>
<td>10 – 20</td>
<td>13</td>
<td>1.25</td>
<td>1.08 – 1.55</td>
<td>9 of 13 (69%)</td>
</tr>
<tr>
<td>1 – 10</td>
<td>44</td>
<td>1.35</td>
<td>1.11 – 1.89</td>
<td>34 of 44 (77%)</td>
</tr>
</tbody>
</table>

* Monthly peaking ratio = (average number of passengers per day during peak month) / (average number of passengers per day during entire year)

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### Estimating Annual Capacity: Generalization

- Let $C$ be the typical saturation capacity per hour of airport $X$ and let
  
  $$A = C \times 24 \times 365 = C \times 8760$$

- Then the annual capacity of $X$ will be in the range of 50%–60% of $A$, the percentage depending on local conditions of use and peaking patterns.

- **Note:** If instead of saturation capacity, $C$ is the declared capacity, then the annual capacity will be in the range of 60%–70% of $A$, since the declared capacity is usually set to approximately 85%–90% of saturation capacity.
Measuring and Attributing Delay

- It is extremely difficult to use field data to measure and attribute delay when congestion is severe
- Tightly inter-connected, complex system
- Users react dynamically to delays (feedback effects, flight cancellations)
- Geographical spreading (no single location for measurement), temporal propagation and secondary effects
- Delay-free, nominal travel times are not readily available
- Causality is unclear

Sequencing and Spacing of EWR Traffic

In the US a flight is counted as “late” if it arrives at the gate more than 15 minutes later than scheduled.

In recognition of habitual delays, airlines have been lengthening the scheduled duration of flights

- improve “on-time arrival” statistics
- improve reliability of their schedules

Thus, a flight that arrives on schedule may in truth have been significantly delayed!
Airfield Delay: A Few Points to Remember

- The relationship between demand and capacity, on one hand, and delay, on the other, is highly nonlinear.
- Serious delays may occur even when average demand is less than (but close to) capacity.
- If demand is close to capacity in good weather, then large delays will occur under worse conditions.
- The dynamic behavior of delay is complex.
- When demand exceeds 85-90% of typical capacity for extended parts of the day, then both average delay and the variability of delay will be large.
- Attribution of delays to specific causes is extremely difficult.
1. The dynamic behavior of a queue can be complex and difficult to predict.
2. Expected delay changes non-linearly with changes in the demand rate or the capacity.
3. The closer the demand rate is to capacity, the more sensitive expected delay becomes to changes in the demand rate or the capacity.
4. The time when peaks in expected delay occur may lag behind the time when demand peaks.
5. The expected delay at any given time depends on the “history” of the queue prior to that time.
6. The variance (variability) of delay also increases when the demand rate is close to capacity.
Example of the Dynamic Behavior of a Queue

Expected delay for four different levels of capacity
(R1 = capacity is 80 movements per hour; R2 = 90; R3 = 100; R4 = 110)

Dynamic Behavior of Queues [2]

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**Key Airport System Flows**

- Arrivals
- Departures
- Pax Screen
- Check-In
- Bag Screen
- Drop-off
- Parking
- Gnd Trans
- Passengers
- Bags/Cargo
- Gnd Trans
- Airside
- Groundside
The Ground Delay Problem

Motivation: If long delays must be suffered, they would be better taken on the ground, prior to take-off

- Safety, workload, fuel

- Must be solved in the presence of uncertainty regarding airport capacity

- “Type 1 Error”
  - The destination airport’s capacity turns out to be lower (or demand to be higher) than forecast, leading to long airborne delays

- “Type 2 Error”
  - The destination airport’s capacity turns out to be lower (or demand to be higher) than forecast, meaning that unnecessary delays were taken on the ground at the airports of origin

Flight Delays: 1995 to 2006

Source: FAA OPSNET data, from Prof. R. J. Hansman, MIT
Case of LaGuardia (LGA)

- Since 1969: Slot-based High Density Rule (HDR)
  - DCA, JFK, LGA, ORD; “buy-and-sell” since 1985
- Early 2000: About 1050 operations per weekday at LGA
- April 2000: Air-21 (Wendell-Ford Aviation Act for 21st Century)
  - Immediate exemption from HDR for aircraft seating 70 or fewer pax on service between small communities and LGA
- By November 2000 airlines had added over 300 movements per day; more planned: virtual gridlock at LGA
- December 2000: FAA and PANYNJ implemented slot lottery and announced intent to develop longer-term policy for access to LGA
- Lottery reduced LGA movements by about 10%; dramatic reduction in LGA delays
- June 2001: Notice for Public Comment posted with regards to longer-term policy that would use “market-based” mechanisms
- Process stopped after September 11, 2001
- FAA, DOT working on problem now ➔ HDR expires 2007

The Airside as a Queuing Network

![Diagram showing the airside as a queuing network with entry fix, arrivals, departures, ATC, and exit fix.]
Flight Delays: 2000 to 2005

National Delays (in minutes)

Source: FAA OPSNET data, from Prof. R. J. Hansman, MIT

Strategic and Tactical ATFM Actions in US

Terminal ATC

en route ATC

PILOTS

TIMU / DISPATCH

Page 60
“Hidden Delay” in US Is Very Large

Comparison of "true" delays vs. other delay measures

<table>
<thead>
<tr>
<th>Year</th>
<th>Ave. Delay relative to schedule (min/op)</th>
<th>Ave. True Delay (min/op)</th>
<th>Ave. Delay relative to scheduled transit time (min/op)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>6.9</td>
<td>11.1</td>
<td>-1.5</td>
</tr>
<tr>
<td>1997</td>
<td>8.8</td>
<td>13.2</td>
<td>-0.3</td>
</tr>
<tr>
<td>2000</td>
<td>11.9</td>
<td>16.9</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

Monthly Delay Statistics (USA)

OPSNET National Delay

- 2002
- 2001
- 2000
- 1999
- 1998

Month

Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
Comparison of August Weekday Peaking Patterns
1993 vs. 1998 (3 Hour Average)

Operations

"Hidden Delay" May Be Very Large

Distribution of Actual Flight Durations
(BOS->DCA, 05-99)
Behavior of Queuing Systems in the “Long Run” [3]

- The relationship between demand and capacity, on the one hand, and airport delay, on the other is very non-linear
  - When demand is close to capacity, small changes in demand or in capacity can cause large changes in delay

- The “proportionality constant” increases with the variability of demand inter-arrival times and of service times
Terminology

**Arrivals to system:**
x = “inter-arrival times” = time between occurrence of successive demands;  E[x];  $\sigma_x^2$

$\lambda$ = “demand rate” = expected no. of customers arriving at queuing system per unit of time

Note:  $\lambda = \frac{1}{E[x]}$

**Service at system:**
t = service time of a “customer”;  E[t];  $\sigma_t^2$

$\mu$ = “capacity” = expected no. of customers who can be served per unit of time

Note:  $\mu = \frac{1}{E[t]}$

Relationships among the Four Measures in Steady-State

$W = W_q + E[t] = W_q + \frac{1}{\mu}$  \hspace{1cm} (1)

[Note: (1) makes full intuitive sense]

$L_q = \lambda W_q$  \hspace{1cm} (2)

$L = \lambda W$  \hspace{1cm} (3)

[Note: (2) and (3) are far less obvious and are known as “Little’s formulae”.]
Four Major Measures of Performance

With system in equilibrium ("steady state"): 

\[ L_q = \text{expected no. of customers in queue} \]

\[ W_q = \text{expected waiting time in queue} \]

\[ L = \text{expected no. of customers in system} \]
\[ (\text{includes those waiting and those receiving service}) \]

\[ W = \text{expected total time in system} \]
\[ (\text{waiting time plus time in service}) \]

An Important Special Case: 
The "P-K formula"

For queuing systems with Poisson demands, 
ANY type of service time, one server and infinite 
queuing capacity (M/G/1 system):

\[ W_q = \frac{\lambda \left[ \frac{1}{\mu} \right]^2 + \sigma_t^2}{2(1 - \rho)} = \frac{\lambda \left[ E^2[t] + \sigma_t^2 \right]}{2(1 - \rho)} \]

Assumes steady-state conditions: \( \rho < 1 \) (\( \lambda < \mu \))
DELAYS model

- Approximates, with high precision, the M(t)/E_k(t)/1 queue
- **Inputs**: Dynamic demand profile (typically specified via hourly demand rates); dynamic capacity profile (typically hourly capacity)
- **Approach**: Starting with initial conditions at time t=0, solves equations describing evolution of queues and computes probabilities of having 0, 1, 2, 3, …. aircraft in queue at times t = Δt, 2Δt, 3Δt, ..... up to end of time period of interest
- **Outputs**: Statistics about queues (average queue length, average waiting time, fraction of flights delayed more than X minutes, etc.)

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Monthly Delay Statistics

![OPSNET National Delays Chart](chart)

Source: FAA
Steps in an Airside Capacity/Delay Analysis

1. Identify all available runway configurations.
2. Compute the (maximum throughput) capacity of each configuration.
3. Prepare the capacity coverage chart for the airport and understand true utilization of various configurations.
4. Develop typical demand profiles for the number of runway movements in a day.
5. Compute delays for typical combinations of demand and available capacity over a day.
6. Draw conclusions based on the above.
Generic queueing system

Source of users → Queue → Service facility

Point of "arrival" at the system

Departure from the system

Server 1
Server 2
Server 3
Server m - 1
Server m

AA DFW shift to rolling banks

Flights per 15-minute period

Time of day

August 2001
March 2003 (scheduled)
Delay Trends

OPSNET National Delays

Thousands of Delays

Month

Delay variability with monthly traffic

Data source: CFMU

Source: Eurocontrol PRC (2001)
Runway Example

- Single runway, mixed operations
- $E[t] = 75$ seconds; $\sigma_t = 25$ seconds
  $\mu = 3600 / 75 = 48$ per hour
- Assume demand is relatively constant for a sufficiently long period of time to have approximately steady-state conditions
- Assume Poisson process is reasonable approximation for instants when demands occur

Estimated expected queue length and expected waiting time

<table>
<thead>
<tr>
<th>$\lambda$ (per hour)</th>
<th>$\rho$</th>
<th>$L_q$</th>
<th>$L_q$ (% change)</th>
<th>$W_q$ (seconds)</th>
<th>$W_q$ (% change)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.625</td>
<td>0.58</td>
<td></td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>30.3</td>
<td>0.63125</td>
<td>0.60</td>
<td>3.4%</td>
<td>71</td>
<td>2.9%</td>
</tr>
<tr>
<td>36</td>
<td>0.75</td>
<td>1.25</td>
<td></td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>36.36</td>
<td>0.7575</td>
<td>1.31</td>
<td>4.8%</td>
<td>130</td>
<td>4%</td>
</tr>
<tr>
<td>42</td>
<td>0.875</td>
<td>3.40</td>
<td></td>
<td>292</td>
<td></td>
</tr>
<tr>
<td>42.42</td>
<td>0.88375</td>
<td>3.73</td>
<td>9.7%</td>
<td>317</td>
<td>8.6%</td>
</tr>
<tr>
<td>45</td>
<td>0.9375</td>
<td>7.81</td>
<td></td>
<td>625</td>
<td></td>
</tr>
<tr>
<td>45.45</td>
<td>0.946875</td>
<td>9.38</td>
<td>20.1%</td>
<td>743</td>
<td>18.9%</td>
</tr>
</tbody>
</table>

Can also estimate PHCAP $\approx 40.9$ per hour
Delays at Chicago O'Hare (ORD)

Source: FAA OPSNET data

Flight Delays Reemerging

Source: FAA OPSNET data
A Simple Case

- Assume $n$ stands; all can accommodate all aircraft sizes
- Subdivide aircraft into $K$ relatively homogeneous classes w.r.t. $SBT$
  \[
  E[SBT] = \sum_{i=1}^{K} p_i \cdot SBT_i
  \]

- Dynamic capacity $= n / E[SBT]$
Punctuality and Delay: Europe vs. US

<table>
<thead>
<tr>
<th></th>
<th>EUROCONTROL Area</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure punctuality</td>
<td>74%</td>
<td>74%</td>
</tr>
<tr>
<td>Source: US BTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average departure delay per</td>
<td>39 minutes</td>
<td>44  minutes</td>
</tr>
<tr>
<td>delayed flight (&gt;15 minutes)</td>
<td>Source: US BTS</td>
<td></td>
</tr>
<tr>
<td>Arrival punctuality</td>
<td>73%</td>
<td>77%</td>
</tr>
<tr>
<td>Source: US BTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average arrival delay per</td>
<td>43 minutes</td>
<td>53  minutes</td>
</tr>
<tr>
<td>delayed flight (&gt;15 minutes)</td>
<td>Source: US BTS</td>
<td></td>
</tr>
</tbody>
</table>

Source: Eurocontrol PRC (2001)

Flight Delays: 1995 to 2005

Source: FAA OPSNET data, from Prof. R. J. Hansman, MIT